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Utilizing a triangular fuzzy chain-ladder in claims reserving of Indonesian general insurance

R A Putri[1] and M D Kartikasari[2]

[1, 2]Department of Statistics, Faculty of Mathematics and Natural Sciences, Universitas Islam Indonesia, Jalan Kaliurang Km 14.5 Sleman, Yogyakarta, Indonesia 55584

E-mail: rizkialifahp@gmail.com, mujiatikartikasari@uii.ac.id

Abstract. Claims reserve is an important issue to be solved regularly in general insurance companies. A chain-ladder is the most of method which is used to calculate claims reserve in general insurance. In the chain-ladder method, the future claim amount or the claim number is predicted with the development factors. However, the development factors contain a vague conclusion. To overcome this, we use a triangular fuzzy chain-ladder which is a combination between fuzzy number and chain-ladder method. We apply this method to the data of vehicle insurance claims from an Indonesian general insurance company.

1. Introduction

To be able to fulfill the liabilities companies, general insurance companies set claims reserve routinely at the moment evaluation. Generally, a calculation of claims reserve is based on a scheme of run off triangle data [1,2]. There are different statistical methods to calculate claims reserve. One of the methods used to calculate claims reserve is the chain-ladder method. A chain-ladder method [3] is the most popular method for calculating claims reserve because the method is simple and reliable when compared to other methods [4].

The claims reserve can be determined by making a calculation from the claim amount or the claim number. Development factors in the chain-ladder method are needed to predict the number of claims. Even though the development factors can be calculated based on the chain-ladder method, actuaries intuitively tend to adjust the pattern of development factors based on their subjective assessment [5]. Therefore, the development factors are not crisp any longer but vague [5]. To overcome a vagueness of development factors, Heberle and Thomas [5] propose a triangular fuzzy chain-ladder method. The triangular fuzzy chain-ladder is a combination of fuzzy set theory [6] and a chain-ladder method. As known, noted that the income premi of motor vehicle insurance has a large contribution in the insurance industry in Indonesia so in this paper, we apply the triangular fuzzy chain-ladder method for data of vehicle insurance claims from an Indonesian general insurance company. The performance of this methods is compared to the original chain-ladder method. Where, in the triangular fuzzy chain-ladder method is obtained the value of "decision maker risk parameter".

Following this section, in the next section we introduce the triangular fuzzy chain-ladder method proposed by Heberle and Thomas [5]. Section 3 gives analysis using the triangular fuzzy chain-ladder method. The conclusion based on the result is presented in section 4.

2. Triangular fuzzy chain-ladder method

The methods derived from a combination of fuzzy number and chain-ladder method are called Triangular Fuzzy Chain-Ladder. The triangular fuzzy chain-ladder method procedure consists of six parts which are described as follows [5]:

Step 1: Arrange a cumulative claims data $C_{i,j}$ in the accident period $i \in \{1,2,\cdots,I\}$ and development period $j \in \{1,2,\cdots,J\}$. Up to time I, the observation data are denoted by $\mathcal{O}_I = \left\{C_{i,j} | i + j \leq I\right\}$. The specification of the data called run-off triangle scheme is demonstrated in Table 1.

Table 1. Scheme of run-off triangle data.

Accident Period	Development Period						
	1	2	$\cdots$	j	$\cdots$	$J-1$	J
1	C_{11}	C_{12}	$\cdots$	C_{1j}	$\cdots$	$C_{1(J-1)}$	C_{1J}
2	C_{21}	C_{22}	$\cdots$	C_{2j}	$\cdots$	$C_{2(J-1)}$	
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$		
i	C_{i1}	C_{i2}	$\cdots$	C_{ij}			
$\cdots$	$\cdots$	$\cdots$	$\cdots$				
$I-1$	$C_{(I-1)1}$	$C_{(I-1)2}$					
I	C_{I1}						

Step 2: Compute the triangular fuzzy chain-ladder factors $\hat{\tilde{f}}_j\left(\hat{f}_j, \hat{l}_{\hat{f}_j}, \hat{r}_{\hat{f}_j}\right)$ by using the following formula

$$\hat{f}_j = \frac{\sum_{i=1}^{I-j-1} C_{i,j+1}}{\sum_{i=1}^{I-j-1} C_{i,j}} \tag{1}$$

$$\hat{l}_{\hat{f}_j} = \hat{r}_{\hat{f}_j} = \frac{\sum_{i=1}^{I-j-1} X_{i,j+1}}{\sum_{i=1}^{I-j-1} C_{i,j}} \tag{2}$$

where

$$X_{i,j+1} = C_{i,j+1} - C_{i,j} \tag{3}$$

for $i \in \{1,2,\cdots,I\}$ and $j \in \{1,2,\cdots,J\}$.

Step 3: Predict the ultimate claims $\hat{\tilde{C}}_{i,J}$ with $i \in \{2,3,\cdots,I\}$ by

$$\hat{\tilde{C}}_{i,J} = \hat{\tilde{C}}_{i,I-1} \prod_{j=I-i}^{J-1} \hat{\tilde{f}}_j \tag{4}$$

where

$$\prod_{j=I-i}^{J-1} \hat{\tilde{f}}_j = \left(\hat{F}_{I-i}^{J-1}, \hat{l}_{\hat{F}_{I-i}^{J-1}}, \hat{r}_{\hat{F}_{I-i}^{J-1}}\right) \tag{5}$$

with

$$\hat{F}_{I-i}^{J-1} = \prod_{j=I-i}^{J-1} \hat{f}_j \tag{6}$$

$$\hat{l}_{\hat{F}_{I-i}^{J-1}} = \hat{F}_{I-i}^{J-1} - 1 \tag{7}$$

$$\hat{r}_{\hat{F}_{I-i}^{J-1}} = \prod_{j=I-i}^{J-1}\left(2\hat{f}_j - 1\right) - \hat{F}_{I-i}^{J-1} \tag{8}$$

for $i \in \{2,3,\cdots,I\}$.

Step 4: Calculate claims reserve for a given accident period $i \in \{1,2,\cdots,I\}$ by

$$\hat{\tilde{R}}_i = \hat{\tilde{C}}_{i,J} - \tilde{C}_{i,I-i} \tag{9}$$

for $i \in \{1,2,\cdots,I\}$ where $\tilde{C}_{i,I-i} = \left(C_{i,I-i}, 0,0\right)$.

Step 5: Compute the aggregated claims reserve by

$$\hat{\tilde{R}} = \sum_{i=1}^{I} \hat{\tilde{R}}_i. \tag{10}$$

Step 6: Determine the expected values of the reserve $E_\beta\left(\hat{\tilde{R}}_i\right)$ with $i \in \{1,2,\cdots,I\}$ for different choices of the "decision-maker risk parameter" β with $0 \leq \beta \leq 1$ by

$$E\left(\tilde{C}_{i,j+1} | \tilde{C}_{i,j}\right) = C_{i,j}\left(\hat{f}_j - \frac{1-\beta}{2}\hat{l}_{\hat{f}_j} + \frac{\beta}{2}\hat{r}_{\hat{f}_j}\right). \tag{11}$$

3. Results

The data used in this paper are data of vehicle insurance claims obtained from general insurance company in Indonesia, with observation period January 2012 to December 2012. The data contains 22,144 claims. The cumulative run-off triangles for the data are given in Table 2.

Table 2. Cumulative run-off triangles (in million rupiahs)

Accident	Development Period											
Period	1	2	3	4	5	6	7	8	9	10	11	12
1	551	2,859	5,018	6,229	6,861	7,548	8,013	8,080	8,478	8,630	8,840	8,903
2	353	3,064	4,547	5,403	6,451	6,865	6,975	7,435	7,663	7,942	8,122	
3	522	2,723	4,711	5,954	6,575	7,050	7,425	7,635	7,784	8,080		
4	218	2,479	4,425	5,403	5,763	6,270	6,613	6,813	6,935			
5	342	2,502	4,150	4,984	5,635	6,254	6,835	7,238				
6	342	2,981	4,591	5,706	6,568	7,375	7,768					
7	294	1,947	3,594	5,160	6,167	6,771						
8	49	758	2,402	3,667	4,302							
9	102	1,494	3,787	4,955								
10	106	1,998	4,153									
11	136	1,713										
12	156											

For the data, the calculated triangular fuzzy chain-ladder factors $\hat{\tilde{f}}_j$ $(j = 1,\cdots,J-1)$ are given in Table 3.

Table 3. Triangular fuzzy chain-ladder factors.

$\hat{\tilde{f}}_j$	Development Period										
	1	2	3	4	5	6	7	8	9	10	11
$\hat{f}_j$	8.1325	1.8143	1.2750	1.1369	1.0934	1.0548	1.0374	1.0300	1.0304	1.0235	1.0071

$\hat{l}_{\hat{f}_j}$	7.1325	0.8143	0.2750	0.1369	0.0934	0.0548	0.0374	0.0300	0.0304	0.0235	0.0071
$\hat{r}_{\hat{f}_j}$	7.1325	0.8143	0.2750	0.1369	0.0934	0.0548	0.0374	0.0300	0.0304	0.0235	0.0071

We are able to predict the unobservable triangle that are located in the lower right part of the run-off triangle given Table 2. By using (4) and calculated triangular fuzzy chain-ladder factors in Table 3, the result of run-off triangle prediction is displayed in Table 4.

Table 4. Cumulative run-off triangle prediction (in million rupiahs).

$\hat{\hat{C}}_{i,j}$	Development Period										
	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{C}_{1,j}$	551	2,859	5,018	6,229	6,861	7,548	8,013	8,080	8,478	8,630	8,840	8,903
$\hat{l}_{\hat{C}_{1,j}}$	0	0	0	0	0	0	0	0	0	0	0	0
$\hat{r}_{\hat{C}_{1,j}}$	0	0	0	0	0	0	0	0	0	0	0	0
$\hat{C}_{2,j}$	353	3,064	4,547	5,403	6,451	6,865	6,975	7,435	7,663	7,942	8,122	8,180
$\hat{l}_{\hat{C}_{2,j}}$	0	0	0	0	0	0	0	0	0	0	0	58
$\hat{r}_{\hat{C}_{2,j}}$	0	0	0	0	0	0	0	0	0	0	0	58
$\hat{C}_{3,j}$	522	2,723	4,711	5,954	6,575	7,050	7,425	7,635	7,784	8,080	8,270	8,329
$\hat{l}_{\hat{C}_{3,j}}$	0	0	0	0	0	0	0	0	0	0	190	249
$\hat{r}_{\hat{C}_{3,j}}$	0	0	0	0	0	0	0	0	0	0	190	252
$\hat{C}_{4,j}$	218	2,479	4,425	5,403	5,763	6,270	6,613	6,813	6,935	7,146	7,315	7,367
$\hat{l}_{\hat{C}_{4,j}}$	0	0	0	0	0	0	0	0	0	211	379	431
$\hat{r}_{\hat{C}_{4,j}}$	0	0	0	0	0	0	0	0	0	211	389	447
$\hat{C}_{5,j}$	342	2,502	4,150	4,984	5,635	6,254	6,835	7,238	7,455	7,682	7,863	7,919
$\hat{l}_{\hat{C}_{5,j}}$	0	0	0	0	0	0	0	0	217	444	625	681
$\hat{r}_{\hat{C}_{5,j}}$	0	0	0	0	0	0	0	0	217	457	659	725
$\hat{C}_{6,j}$	342	2,981	4,591	5,706	6,568	7,375	7,768	8,059	8,300	8,553	8,754	8,817
$\hat{l}_{\hat{C}_{6,j}}$	0	0	0	0	0	0	0	290	532	784	986	1,048
$\hat{r}_{\hat{C}_{6,j}}$	0	0	0	0	0	0	0	290	549	835	1,076	1,153
$\hat{C}_{7,j}$	294	1,947	3,594	5,160	6,167	6,771	7,142	7,409	7,631	7,863	8,048	8,106
$\hat{l}_{\hat{C}_{7,j}}$	0	0	0	0	0	0	371	638	860	1,092	1,277	1,335
$\hat{r}_{\hat{C}_{7,j}}$	0	0	0	0	0	0	371	666	928	1,217	1,459	1,537
$\hat{C}_{8,j}$	49	758	2,402	3,667	4,302	4,704	4,962	5,147	5,302	5,463	5,592	5,631
$\hat{l}_{\hat{C}_{8,j}}$	0	0	0	0	0	402	660	845	999	1,161	1,289	1,329
$\hat{r}_{\hat{C}_{8,j}}$	0	0	0	0	0	402	704	942	1,152	1,384	1,577	1,640
$\hat{C}_{9,j}$	102	1,494	3,787	4,955	5,633	6,159	6,496	6,739	6,941	7,152	7,320	7,373
$\hat{l}_{\hat{C}_{9,j}}$	0	0	0	0	678	1,204	1,542	1,784	1,986	2,198	2,366	2,418
$\hat{r}_{\hat{C}_{9,j}}$	0	0	0	0	678	1,331	1,814	2,193	2,526	2,891	3,195	3,293
$\hat{C}_{10,j}$	106	1,998	4,153	5,296	6,020	6,582	6,943	7,203	7,419	7,644	7,824	7,880

$\hat{l}_{\hat{C}_{10,j}}$	0	0	0	1,142	1,867	2,429	2,790	3,049	3,265	3,491	3,671	3,727
$\hat{r}_{\hat{C}_{10,j}}$	0	0	0	1,142	2,180	3,149	3,855	4,402	4,882	5,405	5,839	5,978
$\hat{C}_{11,j}$	136	1,713	3,108	3,963	4,505	4,926	5,196	5,390	5,552	5,721	5,855	5,897
$\hat{l}_{\hat{C}_{11,j}}$	0	0	1,395	2,250	2,792	3,213	3,483	3,677	3,839	4,007	4,142	4,184
$\hat{r}_{\hat{C}_{11,j}}$	0	0	1,395	3,017	4,385	5,625	6,511	7,192	7,785	8,428	8,959	9,128
$\hat{C}_{12,j}$	156	1,266	2,297	2,929	3,330	3,640	3,840	3,984	4,103	4,228	4,327	4,358
$\hat{l}_{\hat{C}_{12,j}}$	0	1,110	2,141	2,773	3,174	3,485	3,684	3,828	3,947	4,072	4,172	4,202
$\hat{r}_{\hat{C}_{12,j}}$	0	1,110	3,950	6,754	9,003	10,995	12,401	13,471	14,398	15,399	16,223	16,485

Based on calculation of cumulative run-off triangle prediction in Table 4, we can determine claims reserve prediction of vehicle insurance claims $\hat{\tilde{R}}_i$. Table 5 shows the claims reserve prediction for this study. Furthermore, we calculate expected value for the claims reserve prediction $E_\beta\left(\hat{\tilde{R}}_i\right)$ for different choices of the "decision-maker risk parameter" β. The results are shown in Table 6 and compared with the claims reserve prediction from chain-ladder method.

Table 5. Claims reserve prediction (in million rupiahs).

Accident Period	$\hat{\tilde{R}}_i = (\hat{R}_i, \hat{l}_{\hat{R}_i}, \hat{r}_{\hat{R}_i})$		
	$\hat{R}_i$	$\hat{l}_{\hat{R}_i}$	$\hat{r}_{\hat{R}_i}$
1	0.00	0.00	0.00
2	57.92	57.92	57.92
3	249.00	249.00	251.71
4	431.23	431.23	446.70
5	680.59	680.59	724.70
6	1,048.03	1,048.03	1,153.50
7	1,334.78	1,334.78	1,536.95
8	1,329.02	1,329.02	1,639.84
9	2,418.05	2,418.05	3,292.90
10	3,726.66	3,726.66	5,978.29
11	4,183.69	4,183.69	9,128.15
12	4,202.41	4,202.41	16,485.21
Total	19,661.37	19,661.37	40,695.86

Table 6. Expected values of the claims reserve prediction for different choices of the "decision maker risk parameter" and the claims reserve prediction from chain-ladder method (in million rupiahs).

Accident Period	$E_\beta\left(\hat{\tilde{R}}_i\right)$					
	$\beta = 0.1$	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 0.9$	Chain-ladder Reserve
1	0.00	0.00	0.00	0.00	0.00	0.00
2	34.75	43.44	57.92	72.40	81.09	57.92
3	149.53	187.09	249.67	312.26	349.81	249.00

4	259.51	325.35	435.09	544.83	610.68	431.23
5	410.56	515.96	691.62	867.28	972.68	680.59
6	634.09	799.21	1,074.40	1,349.59	1,514.70	1,048.03
7	810.98	1,026.36	1,385.32	1,744.29	1,959.67	1,334.78
8	812.95	1,035.62	1,406.73	1,777.83	2,000.50	1,329.02
9	1,494.57	1,922.89	2,636.76	3,350.63	3,778.95	2,418.05
10	2,348.58	3,076.45	4,289.57	5,502.69	6,230.56	3,726.66
11	2,757.44	3,755.82	5,419.80	7,083.78	8082.17	4,183.69
12	3,135.58	4,687.15	7,273.11	9,859.06	11,410.63	4,202.41
Total	12,848.55	17,375.34	24,919.99	32,464.64	36,991.44	19,661.37

Table 6 shows, the predicted values of claims reserve using triangular fuzzy chain-ladder method for different choices of the “decision maker risk parameter” and the predicted values of claims reserve from the chain-ladder method. In the Triangular Fuzzy Chain-Ladder method “decision maker risk parameter” are the parameters used in determining the best claim reserve value. The result obtained are compared with the chain-ladder method. A choice of “decision maker risk parameter” $\beta \leq 0.5$ gives a fewer predicted value than chain-ladder reserve, and vice versa. As in [5], a choice of “decision maker risk parameter” $\beta \geq 0.5$ indicates risk-free of claims reserve prediction. Therefore, the predicted value of claims reserve in this study with $\beta \geq 0.5$ in Table 6 is preferred. As a result, the predicted value of claims reserve using fuzzy chain ladder method higher than the predicted value of claims reserve using chain ladder method.

4. Conclusion

Triangular fuzzy chain-ladder method is a method derived from a combination of fuzzy number and chain-ladder method. By using this method, we have compared the predicted value of claims reserve for different choices of the “decision maker risk parameter” and the predicted value of claims reserve from chain-ladder method. Based on these comparisons, the predicted value of claims reserve using fuzzy chain ladder method higher than the predicted value of claims reserve using chain ladder method.

References

[1] Pinheiro PJR, E Sliva JMA, and Centeno MDL 2003 Bootstrap Methodology in Claim Reserving *The J. of Risk and Insurance* **70(4)** 701-714.

[2] Antonio K, Beirlant J, Hoedemakers T, and Verlaak R 2006 Lognormal Mixed Models for Reported Claims Reserves *North American Actuarial J.* **10(1)** 30-48.

[3] Mack T 1993 Distribution-Free Calculation of the Standard Error of Chain Ladder Reserve Estimates *ASTIN Bulletin* **23(2)** 213-225

[4] Wuthrich MV and Merz M 2008 *Stochastic Claims Reserving Methods in Insurance* (England: John Wiley & Sons, Ltd).

[5] Heberle J and Thomas A 2014 Combining Chain-Ladder Claims Reserving with Fuzzy Numbers *Insurance: Mathematics and Economics* **55** 96-104.

[6] Zadeh LA 1965 Fuzzy Sets *Information and Control* **8** 338-353.