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How is the Relation: Mathematical Abstraction and Mathematical Connection

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Abstract. As we have known so far, mathematics is a famous subject with its abstractness. If we ask students about their opinions about mathematics and we only give them the opportunity to express them in one word, surely most would argue that mathematics is "abstract". This is because of the lack of understanding of students to connect a mathematical problem with everyday life or vice versa. Therefore, researchers conducted a review of abstraction capabilities and mathematical connection abilities. Based on the results of the review, it was found that abstraction ability was related to mathematical connection skills. In other words, to measure mathematical abstraction capabilities can be supported by instruments of mathematical connection ability.

1. Introduction

During this time we know that mathematics is one of the subjects that is famous for its abstractness [1][2]. The word "abstract" is often interpreted as an expression to express something that is difficult to imagine. In mathematics, abstract objects are often referred to as basic objects, mental objects, or mind objects. The abstract object in question: facts, concepts, operations / relations, and principles [3]. The mathematical object is needed to reorganize previously constructed mathematics vertically to a new structure [4]. Carolyn and Jonathan in 2010 argue that a student who is proficient in mathematics must have the ability of abstraction. This statement asserts that the ability of mathematical abstraction is very important to be owned by students who want to be able to do mathematics.

Abstraction ability in mathematics is defined as the ability of students in the process of constructing mathematical knowledge. This process is essential to understanding students' minds [5] . Ferrari [6] also supports this statement by stating that abstraction is a fundamental process in mathematics. Abstraction in mathematics developed very rapidly, especially around the 20th century. In its development, many experts expressed their opinions on mathematical abstraction. Piaget [7] proposes apart theory of abstraction. According to him, abstraction consists of three, namely: empirical abstraction, pseudo-empirical abstraction, and reflective abstraction. While Cifarelli (1988)[8] argues that abstraction consists of four levels. These levels include the following: the first level is recognition, second level representation, third level structural abstraction, and fourth level structural awareness. Batista and

Clements in 1996 classify abstraction levels from one to four, among others: perceptual, internalization, interiorization, and second level interiorization. Furthermore, Dreyfus [9] argues that abstraction has a sequential level, namely, recognizing, building-with, and constructing.

Gray & Tall [7] define abstraction as a process and concept. The process in question is the process of describing the situation while the concept is the result of the process. In more detail, they translate abstraction into three meanings, namely abstraction is a process, property, and concept. The purpose of these three meanings is abstraction is the process of drawing from a situation and also the concept (the abstraction) output by that process. Based on this opinion, mathematics can have meaning for students because doing the abstraction process students do drawing from the situation. One of the characteristics of this mathematical abstraction is also one of the characteristics possessed by other abilities, namely mathematical connections.

The mathematical connection is one of the important abilities that students must have in learning mathematics. It is proven that mathematical connection capabilities are included in one of the Process Standards in the NCTM. NCTM (The National Council of Teachers of Mathematics) is a forum for teachers to share their aspirations about mathematics education as well as to support teachers and ensure mathematics learning has the highest quality for all students through vision leadership, professional development, and research (Dossey, Halvorsen, & McCrone, 2012). NCTM determines the process standards or abilities expected by students, including abilities: problem-solving, reasoning and proof, communication, connection, and representation [10]. It is seen that the connection is included in the list of process standards that must be in mathematics according to NCTM.

The mathematical connection is the linkage through activities to other concepts, such as basic competence on number and mathematics abilities (Wright, Marthland, & Stafford, 2000). In addition, Noto, Hartono, and Sundawan (Noto, Hartono, & Sundawan, 2016) also expressed similar opinions about mathematical connection abilities, namely the ability to make meaningful relationships between mathematical concepts, between mathematical concepts and other scientific concepts and with everyday life. Based on these two opinions, it is clear that mathematical connection capabilities are also abilities that one of the characteristics is connecting real-world illustrations with the world of mathematics.

Based on this description, an indication of the ability of the abstraction and mathematical connections, the researcher then conducted a study by reviewing how the relationship between mathematical abstraction ability and mathematical connection ability based on the characteristics of both. The review was carried out on published articles.

2. Method

This study is review literature by looking for sources related to abstraction ability and mathematical connection ability. Literature is obtained by searching through search engines from journal providers such as Google Scholar, JSTOR, Mendeley, Elsevier, Scopus, and other journal article providers. The article reviewed is a published article. Each capability is then searched for each of its characteristics to find its connection.

3. Result

3.1 Mathematical Abstraction

Abstraction ability is an important ability in mathematics [11]. Abstraction is a fundamental process in mathematics [6]. The fundamentals referred to are the fundamental mathematical ideas that are based on the investigation of real-world situations and their common key features [12]. However, the importance of abstraction in this mathematical process has not received serious attention. This is evidenced by the lack of studies that discuss abstraction capabilities. Mitchelmore and White [12] argue that the essence of abstraction in mathematics is abstract mathematical objects that take meaning only from the system within which it is defined.

Abstraction has two definitions: abstraction is the process of describing a situation that is a concept of processing result [13]. This statement agrees with Skemp (1986) [12] who argues that abstracting is an activity by which we become aware of similarities. That is, by doing abstraction we can know the similarities of objects or situations that are being abstracted. Then in 1977 Skemp made a basic difference to the fundamental similarities of physical objects and abstractions on the basis of relationships experienced when students manipulated these objects. this statement explains that abstraction has two meanings. The first meaning, abstraction is the process of analyzing the similarities and differences of an object. The second meaning, abstraction is the process of manipulating objects resulting from abstraction based on the relationships on the object. In connection with Skemp's opinion, Vygotsky also stated that "... the foundation of conscious awareness is the generalization or abstraction of mental processes, which leads to their mastery. Instruction has a decisive role in this process. "[14].

Furthermore, Mitchelmore & White [15] also divides abstraction into two, namely empirical abstraction and theoretical abstraction. They also mentioned abstract-apart empirical abstraction and theoretical abstraction as abstract-general. Abstract-apart deals with something that happens in the real world, while abstracts relate to ideas that are common in various contexts. Piaget also divides abstraction into two, namely simple abstraction (experimental or empirical) and reflective abstraction (logical or mathematical). Simple abstraction is an abstraction that is based on the object itself, for example rough, smooth, color, and so on. Reflective abstract is an abstraction that is based on coordination, relations, operations, and uses that do not directly come out from the properties of the object, for example as many as five marbles, when put in a fixed bag of five or fixed position five. Some experts cite reflective abstraction as a theoretical abstraction [12].

According to Skemp [1], empirical abstraction begins with the sensitivity of students to the same characteristics then classifies based on these characteristics. This abstraction allows students to get to know new experiences. Based on the opinions of previous experts, then there are several researchers who also examined the ability of abstraction with various indicators. Some researchers along with abstraction indicators are as follows.

Table 1. The indicator of mathematical abstraction abilities

Author	Year	Indicator
Tata	2015	Reflective abstract: a. Integration and problem formulation b. Transform problems into symbolic forms Emphirical abstrction: a. Make generalizations b. Formation of mathematical concepts related to other concepts c. The formation of further mathematical objects d. Formalization of mathematical objects Theoretical abstract a. The process of manipulating symbols
Rizka & Lukman	2017	a. Identify the characteristics of objects through direct experience b. Identify the characteristics of objects that are manipulated or imagined c. Present mathematical ideas in languages and mathematical symbols d. Apply the concept to the appropriate context.

Nurhasanah, Kusumah, & Subandar	2017	Empirical Abstraction a. Identify the characteristics of objects through direct experience b. Identify the characteristics of manipulated objects or objects that can be imagined. Theoretical abstract a. Represent objects in the form of symbols or mathematical languages b. Make connections between processes or concepts to form new understandings c. Eliminating the material nature of the object, applying the concept to the context appropriately d. Make generalizations e. Manipulate abstract mathematical concepts f. Make connections between processes and concepts to shape new knowledge.

3.2 Mathematical Connection

The mathematical connection is one of the important abilities that need to be mastered by students, especially in mathematics learning. This mathematical connection is divided into two interrelations, namely internal and external linkages. Internal linkages are interrelationships between mathematical concepts, while external linkages are a link between mathematical concepts and everyday life [10]. In line with this statement, [16] also suggests that mathematical connections can be divided into two important points. That opinion is, "The connections standard has two separate thrust. First, it refers to connection within and among mathematical ideas. "This statement should indirectly suggest that mathematical concepts are more often applied to other fields of study or in everyday life. Thus the usefulness of mathematics subjects becomes increasingly more meaningful.

Desforges [17] argues that mathematical connections are the circumference through activities to other concepts, such as basic competence on number and mathematics abilities. Mathematical connection ability is one of the important abilities for students to have because this ability is also included in the mathematics learning goals contained in Permendiknas No 22 of 2006. The National Education Minister said the objectives of mathematics learning are as follows:

a. Understanding mathematical concepts, explaining the interrelationships between concepts or logarithms in a flexible, accurate, efficient, and precise way to solve problems.
b. Using reasoning on patterns and traits, making mathematical manipulations in making conclusions, compiling evidence, or explaining ideas and statements mathematically.
c. Resolve problems that include the ability to understand problems, design mathematical models, analyze models to get results, and interpret results.
d. Communicate ideas with symbols, tables, diagrams, or other media to clarify a condition or problem.
e. Have an attitude of respecting the benefits of mathematics in life, such as having curiosity, attention, and interest in learning mathematics and tenacity and confidence in solving problems.

The importance of mathematical connection skills is also supported by NCTM [18] which states, "When students can connect mathematical ideas, their understanding is deeper and more lasting. In contexts that relate mathematics to other subjects and their own interests and experiences. In this statement, it can be understood that mathematical connection skills can make mathematics more meaningful for students because the understanding becomes deeper and longer lasting.

When making connections, it is just like learning to construct understanding. This is in agreement with Haylock & Tangata in 2007 which states that "the relationship between concrete experiences, language, picture, and mathematical symbols. "Based on the statement, it can be seen that mathematical connections are also very important in the learning process to construct students' understanding.

Bell [19] states that awareness of the need for mathematical connections is also very important in making mathematical connections. When examined further, there is no stand-alone concept in mathematics and no connection with other concepts. Students who in learning practice their ability to connect, then the mathematical concepts that are being studied will become easier to understand. This happens because students are directly involved in finding relationships between concepts so that learning is more meaningful and can be stored longer in the memory of students. This statement is in line with the NCTM described above.

Connections between processes and concepts in mathematics are abstract objects because the connection process occurs in the minds of students. The connection that occurs is connecting between the symbol and its representation. Based on the opinions of previous experts, then there are several researchers who also examined the mathematical connection ability with various indicators. The opinions of some researchers along with mathematical connection indicators are as follows.

Table 2. indicator of mathematical connection ability

Author	Year	Indicator
NCTM	2000	a. Recognize and use connections among mathematical ideas. b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. c. Recognize and apply mathematics in the context outside of mathematics.
Sugiman	2010	a. Inter-connection between mathematical topics, so what is linked is between concepts or principles in the same mathematics topic b. The connection between topics with one topic but still within the scope of mathematics, c. The connection between mathematical concepts and other fields of science d. The connection between mathematical concepts and students' daily lives.
Sukmaningthias	2017	a. Know the relationship between concepts that exist in mathematics b. Understand the relationship between one concept and another c. Using the relationship between concepts in mathematics to solve the problem of mathematical applications.
Katuche	2018	a. Mention the mathematical concepts needed in solving problems related to mathematical concepts b. Mention the mathematical concepts needed to solve problems related to the real world context c. Use the relationship between mathematical concepts and other fields of science in solving mathematical problems d. Mention the relationships and mathematical procedures needed in solving mathematical problems related to the real world.

4. Analyze: The relation about Mathematical Abstraction dan Mathematical Connection Ability

Based on the explanation of abstraction abilities and mathematical connection skills above, it can be seen that both of these abilities have very close links. This is supported by several expert opinions. An extreme opinion was stated by Noss & Hoyle [20] who stated that abstraction as a form connection. This statement reveals that abstractions have the same elements as connections. Mitchelmore & White [12] also mentions that all mathematics has come back to reality links. Though previously explained by Ferrari [6] that abstraction is a fundamental process in mathematics. Based on the description, it can be explained that all mathematics has a relationship or connection with reality, whereas mathematics has a fundamental element, namely abstraction. Thus it can be concluded that abstraction ability has a relationship with mathematical connection ability.

5. Conclusion

Based on some literature that has been reviewed by researchers, it can be concluded that the ability of mathematical abstraction has a very close relationship with mathematical connection skills. This is supported by Ferrari's opinion [6] which states that abstraction is a fundamental process in mathematics. In addition, the opinion of Noss & Hoyle [20] stated that abstraction as a form connection. In learning mathematics, students must learn to acquire abstraction skills even though sometimes they do not realize it. When carrying out the abstraction process, they are also indirectly connecting between problems in the real world and mathematical symbols. These mathematical symbols are the result of the abstraction process.

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