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Mathematical Model Construction of Mass Conservation Law`s Derivative to Reduce the Traffic Congestion in Yogyakarta

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Abstract. Indonesia is a developing country which the average of daily traffic flow is crowded. This is why the density of traffic flow becomes a serious problem and it is the root of the congestion due to the density of the vehicles congestion especially in the city areas getting worse. To realize the highway`s construction, it needs to watch the road wide`s needed and the traffic signs placement. To set the layout of the city it needs a certain calculation so the proportion of vehicle with the traffic is balanced and that will reduce the density of traffic flow that impact on the decline in traffic congestion in Yogyakarta. It appears that the traffic flow is influenced greatly by the density and speed of the vehicle. If the density is maximum and the speed also maximum, there will be crash between vehicles. From the decrease of the conservation of mass formula, the equation of the traffic flow model derived from equilibrium laws that will be discussed in this paper. From this model, the decrease of the traffic congestion problem is possible when considering some models of traffic flows.

1. Introduction

Traffic congestion becomes a serious problem which is being considered by various circles. Density of the traffic flow is actually the root of the congestion due to the density of both personal and public vehicles that continues to grow as the time goes by, which makes the congestion is getting worse particular on narrow and branched. From those conditions, the most important thing to do is to make a plan for the design of roads construction that play a role in traffic management in order to create a regular traffic path, smooth, and barrier-free that makes the riders and other passengers feel comfortable.

To realize the highway construction, it is important to estimates the road`s width and the traffic signs setting. To arrange the layout of the city, it needs a certain calculation so that the proportion of vehicles balanced with the traffic flow. So, that will reduce the traffic congestion in Indonesia because of the reduction of traffic density.

2. Theoretical Review

2.1. Vehicle Density

The density denoted by (ρ) is stating the number of vehicles per kilo-meter on the road [1]. In certain unit time intervals, e.g. area S_1, ρ can be searched by calculate the number of vehicles per partition of the length Δx, mathematically can be written: $\rho = \frac{n}{\Delta x}$. By n is the number of vehicles that pass along the road interval Δx.

2.2. Vehicle Flow Rate

Flow rate is the number of vehicles that pass a road at a certain time interval [1]. For example, at intervals of time ΔT and at location x_2, the flow rate can be searched by: $q = \frac{m}{\Delta T}$. With the index m is the number of vehicles that pass on the location x_2.

2.3. Speed

Speed v is the result between the flow rate with the density. In other words, speed is a function of the location, time, and size of its interval. Mathematically can be written:

$$v = \frac{q}{\rho} = \frac{total\ distance}{total\ of\ time}$$

2.4. Relationship of Vehicle Density, Vehicle Flow Rate and Speed Against Traffic Flow

To provide an explanation of the relationship of these three variables, suppose there are cases if they have a movement of vehicles with constant speed v with density (ρ). Since each state drove at the same rate, the distance between each vehicle is constant, so the traffic density is unchanged. To measure the flow of traffic on the t hour, we can use the formula $s = v \times t$ so that at the time t hour each vehicle will take distance vt. So, the number of passed and observed vehicles by the t hour is a number of vehicles at the distance vt. Since ρ is the number of per kilometer vehicles and the actual distance is vt kilometers, then ρvt is a number of passed and observed vehicles during t hours. So, the number of per hour vehicles or called the traffic flow q can be expressed by the following formula: $q = \rho v$. Since the traffic variables depend on distance and time, the data is written: $q(x,t) = \rho v$.

3. Discussion

This session will describe the stream of traffic flow model in order to reduce the congestion in Yogyakarta. The traffic flow model is derived from the laws of equilibrium as well as the law of conservation of mass which requires that the changes of (kg/s) is $difference\ massa\ per\ time = input\ of\ massa - output\ of\ massa$. If m indicates the number of vehicles which crossing the road, ρ indicate the density, and q is the flow rate of the vehicle, which q_x indicates the flow rate of the vehicle that entering a road while $q_{x+\Delta x}$ is the flow rate of the vehicle coming out of the road, so it can be expressed by the following formula: $\frac{\partial m}{\partial t} = q_x - q_{x+\Delta x}$ (1). Based on the density formula (ρ) where: $\rho = \frac{m}{V}$ for m= mass and V= volume,then $m = \rho V$. Therefore, the equation (1) can be written: $\frac{\partial \rho V}{\partial t} = q_x - q_{x+\Delta x}$ (2). Because the object of discussion is the highway and only one dimension, then the intended volume is the length of the road interval that is equal to Δx, so the equation (2) becomes: $\frac{\partial \rho \Delta x}{\partial t} = q_x - q_{x+\Delta x}$. By dividing the two segments by Δx then: $\frac{\partial \rho}{\partial t} = \frac{q_x - q_{x+\Delta x}}{\Delta x}$.

When $\Delta x \to 0$, $\lim_{\Delta x \to 0} \frac{q_x - q_{x+\Delta x}}{\Delta x} = \frac{\partial \rho}{\partial t}$ so the equation is generated as follows: $\frac{\partial \rho}{\partial t} = -\frac{\partial q}{\partial x}$, Because $q = \rho v$, where v is the speed, then: $\frac{\partial \rho}{\partial t} = -\frac{\partial \rho v}{\partial x}$ so produced the vehicle density model as follows: $\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} = 0$. In the second phase, there will be analysis the relationship of density with velocity by the assumption no other influence factors that so the other ones are ignored. So it can be written: $v = v(\rho)$.

Assumed that there is no other vehicle passing the interval (a,b) means that $\rho = 0$ then the vehicle will go at maximum speed, that is v_{maks}, but if there is an increase in density then the speed rate becomes slow, so it can be written: $v(\rho) \leq 0$. Meanwhile, if the vehicle`s density is maximum (ρ_{maks}), then the road conditions become compressed, in which case the vehicle cannot run or stop. When it is forced, the vehicle will collide, because there is no room for move. So it can be written: $v(\rho_{maks}) = 0$. So, $\rho_{maks} = \frac{1}{L}$ with L is the vehicle`s length. Therefore the relationship between density and velocity can be illustrated by the following figure:

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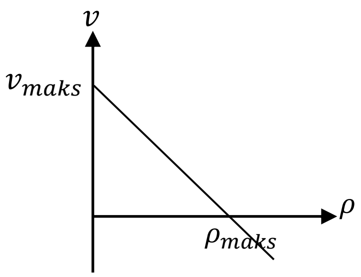

Figure 1. Graph of density and velocity

Based on the figure above, we obtained two points namely two points namely $(0, v_{maks})$ and $(\rho_{maks}, 0)$ so the line equation`s form can be written:

$$\frac{v}{v_{maks}} = \frac{\rho - \rho_{maks}}{-\rho_{maks}}$$

$$v = \frac{-\rho v_{maks}}{\rho_{maks}} + v_{maks}$$

So, we get the density-dependent velocity model in the form of the formula:

$$v(\rho) = v_{maks}\left(1 - \frac{\rho}{\rho_{maks}}\right), 0 \leq \rho \leq \rho_{maks}$$

Once the constructed model above is obtained, a relevant scaling of the model above will be determined. The scale is the size comparison on the model with the actual size, either changing to the smaller size or the larger one without eliminating its characteristics. It will be taken variables θ and μ, where θ indicates distance and μ indicates time. If $v_{maks} = \frac{\theta}{\mu}$ so there will be $x_s = \frac{x}{\theta}$, $t_s = \frac{t}{\mu}$ such that:

$$\rho_t = \frac{\partial \rho}{\partial t_s} \cdot \frac{\partial t_s}{\partial t}$$

So, ρ_t means the density of the traffic flow with independent variable time (t).

4. Numerical Simulation

Traffic flow model`s simulation using Matlab program by taking $\Delta x = 1m$, $\Delta t = 0.01s$, then $v = 3m/s$. So the density change can be seen as follows:

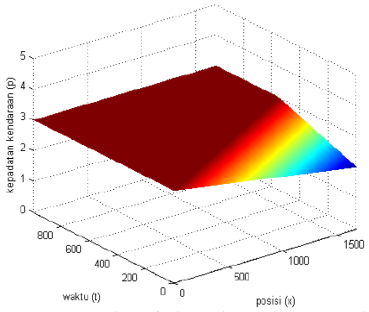

Figure 2. Graph of density changes in case 1

From the picture above can be seen that all the time t on the left limit density of 3 vehicles per unit area and on the right border density 2 vehicles per unit area. This indicates that in the position $x = 0$ there is always 3 vehicles, and when $x = 1680$ there is always 2 vehicles. While to know the condition of density in every position x at a certain t moment then depicted in the following graph:

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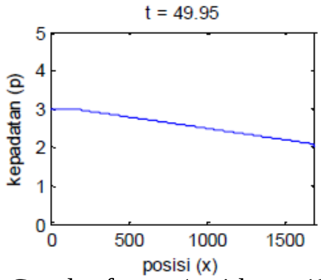

Figure 3. Graph of case 1 with t = 49.95

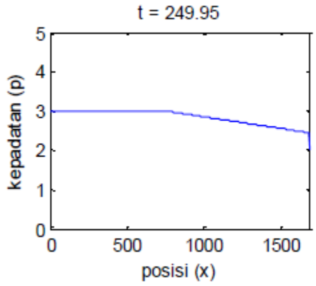

Figure 4. Graph of case 1 with t = 249.95

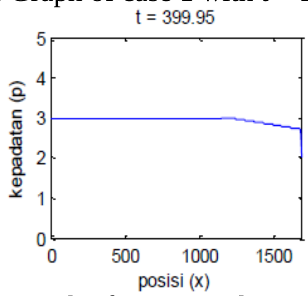

Figure 5. Graph of case 1 with t = 399.95

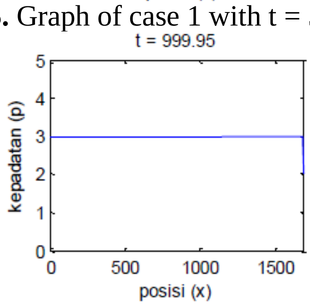

Figure 6. Graph of case 1 with t = 999.95

Based on the graphs above, it is known that at $t = 0$ the vehicle`s density of 2 vehicles per unit area, after $t = 49.95$ the vehicle began to increase. Because the number of vehicles in the left margin more than the right border, so the vehicle piled up at the starting points. While for t go up to infinity the density along the road with the same width, which is 3 vehicles per unit area, but at the end of the road there are still 2 vehicles per unit area.

Simulation in the second case, if it is assumed that the vehicle`s density in the boundary area is the opposite of the first case. It there are 2 vehicles then by doing the same will get the density graph as follows:

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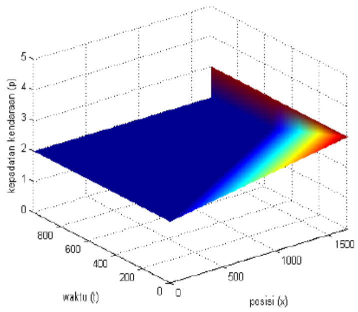

Figure 7. Graph of density changes in case 2

In this second case, the vehicle`s density on the left boundary is 2 vehicles per unit area, while the density on the right limit is 3 vehicles per unit area, so it can be seen that the accumulation of vehicles occurs at the end of the interval will be congested. But once $t = 1000$ the road conditions have reached a balanced density, it means the density is same along the road that is 2 vehicles per unit of road area. As shown in the following graph:

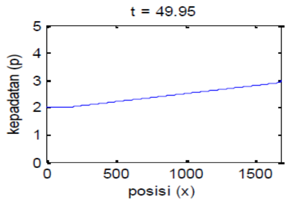

Figure 8. Graph of case 1 with t = 49.95

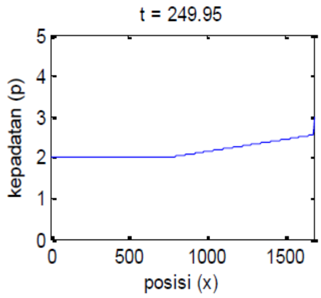

Figure 9. Graph of case 1 with t = 249.95

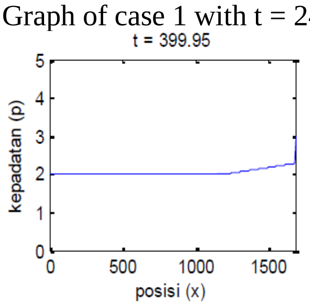

Figure 10. Graph of case 1 with t = 399.95

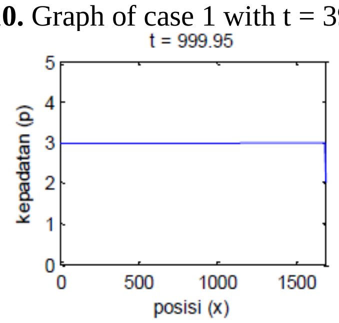

Figure 11. Graph of case 1 with t = 999.95

5. Offered solution

Based on the discussion above, it can be concluded that the model of traffic flow taken from the laws of equilibrium are as follows: $\frac{\partial\rho}{\partial t}+\frac{\partial\rho v}{\partial x}=0$, where ρ is the density and v is the speed of the vehicle. The reason for the laws of equilibrium with conservation of mass is to obtain a balance between the amount of vehicle`s density in a particular road and the speed variable. From the model above, it is known that the traffic flow will be in equilibrium state if $\frac{\partial\rho}{\partial t}=-\frac{\partial\rho v}{\partial x}$ which the amount of change in density over time equals the negative of the changes in the average velocity`s number multiplied by the change in the density of the road interval`s length. So the solution to reduce the traffic congestion is to predict the change in density per time should be proportional to the negative of the average speed times the density changes over the road interval`s length. It appears that the traffic flow is influenced greatly by the density and vehicle`s speed. If $\frac{\partial\rho}{\partial t}+\frac{\partial\rho v}{\partial x}\neq 0$, there is a possibility that the situation of the road will be in chaos where $v(\rho_{maks})=0$, which means the vehicles are stopped, and if it is forced there will be a crash. So, another way to avoid traffic congestion is to maintain $v(\rho_{maks})\neq 0$. To preserve $v(\rho_{maks})\neq 0$, we use the formula $v(\rho)=v_{maks}(1-\frac{\rho}{\rho_{maks}})$. Because $v_{maks}=\frac{\theta}{\mu}$, then $v_{maks}=\frac{xt_s}{tx_s}$, so $v(\rho)=\frac{xt_s}{tx_s}(1-\frac{\rho}{\rho_{maks}})$.

Suppose $u=1-\frac{\rho}{\rho_{maks}}$. Then $\frac{\rho}{\rho_{maks}}=1-u,\ \rho=(1-u)\rho_{maks}$, so $v((1-u)\rho_{maks})=\frac{xt_s}{tx_s}(1-\frac{(1-u)\rho_{maks}}{\rho_{maks}})$. Because $0\leq\rho\leq\rho_{maks}$, then $v(\rho)=\ v((1-u)\rho_{maks})\leq v(\rho_{maks})$, so another way to avoid congestion can be seen from the value of $v(\rho_{maks})$ by maintaining the value of $v(\rho_{maks})\neq 0$. To get the result, then the value can be seen in the acquisition of the last model:

$$\frac{xt_s}{tx_s}\neq\frac{(1-u)\rho_{maks}}{\rho_{maks}}$$

Thus, the traffic congestion problem will decrease more when considering some of the traffic flow models above.

6. Conclusion

From the derivation of the conservation of mass formula, the equation of the traffic flow model derived from the equilibrium laws that is $\frac{\partial\rho}{\partial t}+\frac{\partial\rho v}{\partial x}=0$, where ρ states the vehicle`s density, $v$ states the vehicle`s speed. It appears that the traffic flow is influenced greatly by the density and speed of the vehicle. Based on the model, there is a possibility that the state of the road will be in chaos where $v(\rho_{maks})=0$. For that, a way that can be done to avoid traffic jams is by keeping $v(\rho_{maks})\neq 0$. To maintain $v(\rho_{maks})\neq 0$, it used the formula $v(\rho)=v_{maks}(1-\frac{\rho}{\rho_{maks}})$, so the problem of traffic congestion will decrease when considering some models of traffic flow.

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