Solving the Generalized Trapezoidal Fuzzy Number Linear Programming Problem

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**Abstract.** This research was aimed to determine a ranking function of generalized trapezoidal fuzzy number and the optimum solution of a generalized trapezoidal fuzzy numbers linear programming problem using generalized fuzzy simplex method. The results of this research: we have defined a ranking function of thegeneralized trapezoidal fuzzy numbers is . Based on this definition We have constructed an algorithm to solve the generalized trapezoidal fuzzy numbers linear programming problem that is called as generalized fuzzy simplex method.

**Keywords**: generalized fuzzy simplex method, generalized trapezoidal fuzzy number, ranking function

1. Introduction

Optimization is a process to find the optimal solution of an objective function by considering the existing constraint functions [1]. In this case, the optimization problem can be solved by using a linear programming. Linear programming is an optimization model to maximize or minimize an objective function with linear limits on the variables [2]. Linear programming consists the objective function, main constraint function, and non negative constraint function.

The purpose of the optimization case in this research is to maximize the objective function by considering several constraint functions. In fact, not all of the constraints of the objective function in the linear programming model can be met [3]. In this case, a linear programming will be used with the coefficients and variables in the form of fuzzy numbers. This linear programming was first introduced by Zimmermann which is called the fuzzy linear programming. Fuzzy linear programming is a linear programming method which is applied in fuzzy sets. A fuzzy set is a set whose members have membership values / degrees in the form of intervals from 0 to 1.

During its development, several researchers have developed various types of linear fuzzy programming and various types of approaches in order to get the optimal solution, one of which is Karyati, Wutsqa, & Insani [4] modified Maleki's ranking function [5] to solve the linear trapezoidal fuzzy number programming model. This study uses a trapezoidal fuzzy number with the highest membership function is 1. However, in this case a generalization of trapezoidal fuzzy numbers and a ranking function will be developed in a generalized trapezoidal fuzzy number program by following the rules that have been studied. [6].

1. Preliminaries
2. *Fuzzy Sets*

 In real life, not all problems are clearly defined or can be stated in a crisp set [7]. For example, smart student associations, rich people associations, old people associations, short people associations, etc. To solve this problem, we can used a fuzzy set. Fuzzy set theory is an extension of crisp set theory. A fuzzy set is a set whose the element have a membership degree in the form of real numbers in interval [0,1].

Let is a generalized trapezoidal fuzzy number, the graph of generalized trapezoidal fuzzy number is represented in Figure 1:



**Figure 1.** The Graph of Generalized Trapezoidal Fuzzy Number

Fuzzy number is called a generalized trapezoidal fuzzy number if its membership function satisfies the following characteristics [6]:

1. stricly increasing on and stricly decreasing on
2. for all , where

Thus, a fuzzy number can be said to be a generalized trapezoidal fuzzy number if it fulfills the following membership functions:

1. *Generalized Trapezoidal Fuzzy Number Linear Programming Model*

Let is a set of all fuzzy numbers and R is the set of real numbers, the general form of the generalized trapezoidal fuzzy number linear program model is:

Maximize

Such that

where , , ,

Arithmetic operations on trapezoidal fuzzy numbers have been studied by Karyati, Wutsqa, & Insani [4]. In this case, the arithmetic operations of the trapezoidal fuzzy number are generalized according to the described rules [6]. Let and is a generalized trapezoidal fuzzy number and , then:

1. , for
2. , for .
3. *Ranking Function of Generalized Trapezoidal Fuzzy Number*

In this paper, a ranking function will be used to solve generalized simplex fuzzy in generalized trapezoidal fuzzy number linear programming problem. A ranking function can be obtained using an integral value [8]. Let is a generalized trapezoidal fuzzy number membership function when the graph stricly increasing and is a generalized trapezoidal fuzzy number membership function when the graph stricly decreasing, then the ranking function of generalized trapezoidal fuzzy number is [9]:

As we known that

the inverse of L (x) and R (x) are

so we have

Thus, the generalized trapezoidal fuzzy number ranking function is:

**Theorem 1**

*The ranking function is defined as a mapping .. For all generalized trapezoidal fuzzy numbers and in , defined the following relationship:*

1. Result and Discusions
2. *Generalized Simplex Fuzzy Method*

Generalized simplex fuzzy method is the classic simplex method with optimization testing using the ranking function. The complete generalized fuzzy simplex algorithm is given as follows:

1. Transform the generalized trapezoidal fuzzy number program linear problem into canonical form.
2. Create an initial generalized fuzzy simplex tableau.
3. Optimization test of the generalized trapezoidal fuzzy number linear program using the generalized fuzzy simplex method can be seen in the row where is the ranking function and *j* is the simplex table column.
4. Fix the generalized fuzzy simplex tables can be done by entering non-basis variables with values smallest and leave the base variable which has the ratio smallest positive where .
5. Do elementary row operations on the new table as needed to change the pivot column to the base with a value of 1 on the pivot element. In this case, divide all the elements on the pivot row by the pivot elements. Then change all the other elements in the pivot column to 0 by subtracting the other rows from the pivot rows.
6. Repeat step (3) until all value , so the optimal solution is obtained.
7. *Numerical Simulation*

Based on the problem solving algorithm of generalized trapezoidal fuzzy number linear program using generalized fuzzy simplex method. There are each example of calculating the optimal solution of a generalized trapezoidal fuzzy number linear program of a numerical problem and its solving process.

*Example 1.*

Maximize
such that

The solving steps of Example 1 were carried out using the generalized fuzzy simplex method as follows:

1. Transform the equation into canonical form, so we have

Maximize

such that

1. Solve with generalized fuzzy simplex method

Table 1. Initial Table of Example 1

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  | 2 | 6 | 1 | 0 | 24 | 4 |
|  |  | 7 | 3 | 0 | 1 | 21 | 7 |
|  |  | 0 | 0 |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  | -3,75 | -4 | 0 | 0 |  |  |

Table 2. Second Iteration Generalized Fuzzy Simplex of Example 1

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  | 0,33 |  | 0,16 |  | 4 | 12,01 |
|  |  | 6 |  | -0,5 |  | 9 | 1,5 |
|  |  | (2.3, 3.3, 1.3, 1.6; 0.5) |  | (1.16, 1.6, 0.6, 0.3;0.5) |  | (28, 40, 16, 8; 0.5) |  |
|  |  | (-6.7, -2.7, 3.3, 3.6; 0.5) |  | (1.16, 1.6, 0.6, 0.3;0.5) |  |  |  |
|  |  | -2,312 |  | 0 |  |  |  |

Table 3. Third Iteration Generalized Fuzzy Simplex of Example 1

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | (6,9, 2,2;0.8) | (7,10, 4,2;0.5) |  |  |  |  |
|  |  |  |  |  |  |
| (7,10, 4,2;0.5) |  | 0 |  | 0,19 | -0,056 | 3,5 |  |
| (6,9, 2,2;0.8) |  | 1 | 0 | -0,08 | 0,1667 | 1,5 |  |
|  |  | (6,9, 2,2;0.5) | (7,10, 4,2;0.5) | (0.62, 1.45, 0.93, 0.54; 0.5) | (0.44, 1.11, 0.38, 0.44; 0.5) | (33.5, 48.5, 4.17, 10; 0.5) |  |
|  |  | (-3, 3, 4, 4; 0.5) | (-3,3, 6,6;0.5) | (0.62, 1.45, 0.93, 0.54; 0.5) | (0.44, 1.11, 0.38, 0.44; 0.5) |  |  |
|  |  | 0 |  | 0,468 | 0,395 |  |  |

Based on Table 3, we get the solution for with is

*Example 2.*

Maximize

such that

The solving steps of Example 2 were carried out using the generalized fuzzy simplex method as follows:

1. Transform the equation into canonical form, so we have

Maximize

such that

1. Solve with generalized fuzzy simplex method

**Table 4.** Initial Table of Example 2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | (6,9,2,4;0.75) | (1,3,1,3;0.8) |  |  | (-*M, -M, M, M*;1) |  |  |
|  |  |  |  |  |  |  |
|  |  | 3 | 3 | 1 | 0 | 0 | 12 | 4 |
| (-*M, -M, M, M*;1) |  | 4 | 12 | 0 | -1 | 1 | 24 | 2 |
|  |  | (-4*M, -4M, 4M,* *4M*;1) | (-12*M, -12M,* *12M, 12M*;1) |  | (*M, M, M, M*;1) | (-*M, -M, M, M*;1) |  |  |
|  |  | (-4*M* -9, -4*M* -6, 4*M* +4, 4*M* +2; 0.75) | (-12*M-3, -12M-1,* *12M+3,* *12M+1*;0.8) |  | (*M, M, M, M*;1) | (0*, 0, 2M, 2M*;1) |  |  |
|  |  | -3*M*-6 | -9*M*-2.25 | 0 | *0,75M* |  |  |  |

**Table 5.** Second Iteration Generalized Fuzzy Simplex of Example 2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | (6,9,2,4;0.75) | (1,3,1,3;0.8) |  |  | (-*M, -M, M, M*;1) |  |  |
|  |  |  |  |  |  |  |
|  |  | 2 | 0 | 1 | 0,25 | -0,25 | 6 | 3 |
| (1,3,1,3;0.8) |  | 0,33 | 1 | 0 | -0,083 | 0,083 | 2 | 6,06 |
|  |  | (0.33,1,0.33,1;0.8) | (1,3,1,3;0.8) |  | (-0.249, -0,083, 0.249, 0,083;0.8) | (0.083, 0.249, 0.083, 0.249;0.8) |  |  |
|  |  | (-8.67, -5, 4.33, 3;0.75) | (-2,2,4,4;0.8) |  | (-0.249, -0,083, 0.249, 0,083;0.8) | (0.083+*M*, 0.249+*M*, 0.083+*M*, 0.249+*M*;0.8) |  |  |
|  |  | -5,3756 | 0 | 0 | -0,1556 | 0,1556+0,75*M* |  |  |

**Table 6.** Third Iteration Generalized Fuzzy Simplex of Example 2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | (6,9,2,4;0.75) | (1,3,1,3;0.8) |  |  | (-*M, -M, M, M*;1) |  |  |
|  |  |  |  |  |  |  |
| (6,9,2,4;0.75) |  | 1 | 0 | 0.5 | 0,125 | -0,125 | 3 |  |
| (1,3,1,3;0.8) |  | 0 | 1 | 0,167 | -0,125 | 0,125 | 1 |  |
|  |  | (6,9,2,4; 0.75) | (1,3,1,3; 0.8) | (3.167, 5.001, 1.167, 2.501; 0.75) | (0.375, 1, 0.625, 0.625; 0.75) | (-1, -0.375, 0.625, 0.625; 0.75) |  |  |
|  |  | (-3, 3, 6, 6; 0.75) | (-2, 2, 4, 4; 0.8) | (3.167, 5.001, 1.167, 2.501; 0.75) | (0.375, 1, 0.625, 0.625; 0.75) | (-1+*M*, -0.375+*M*, 0.625+*M*, 0.625+*M*; 0.75) |  |  |
|  |  | 0 | 0 | 3,313 | 0,515 | 0,75M-0,515 |  |  |

Based on Table 3, we get the solution for Example 2 where is

1. Conclusions

Based on the discussion that has been written, it is concluded that by using the definition of the integral value, a ranking function is obtained from generalized trapezoidal fuzzy numbers is .

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