Student difficulties in mathematizing algebra problems

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**Abstract**. Students have difficulties in solving algebra problems. Algebra problems can be solved by mathematizing. There are two categories of mathematization, namely horizontal and vertical mathematization. This study aims to find out the types of difficulties in solving algebra problems and to classify these difficulties according to the mathematization phase. The student difficulties were obtained by synthesizing several articles from previous research using the systematic literature review method. The selected articles must have the following criteria: (1) the topic of articles is relevant to difficulties in mathematizing algebra problems, (2) the articles were published in online's journal from 2009 to 2020, and (3) the research subject is students on formal education from primary school to college. Types of difficulties that obtained are then classified according to the mathematization phase: (1) understand the problems, (2) formulate mathematical models, (3) mathematical problem solving, and (4) reflection. The student difficulties can be observed from student’s errors in solving algebra problems in the dimensions of factual, conceptual, procedural, and metacognitive knowledge. Therefore, the teacher can design appropriate algebra learning by identifying student difficulties.

1. Introduction

Mathematics is a universal language whose rules have been agreed upon by mathematicians all over the world [1]. Students study mathematics from elementary school to college [2]. Students can have more rational and critical thinking patterns when mastering mathematical concepts well [3]. Abstract mathematical concepts make students think that mathematics is something difficult. The teacher needs to present these abstract concepts in the form of contextual problems. Giving contextual problems will make students know the various benefits of learning mathematics. That way, students will realize that mathematics can be a tool in solving everyday problems [1], [2].

Students must learn five math content, namely number and operations, algebra, geometry, measurement, and data analysis and probability. The objectives of students learning algebra are: (1) understanding patterns, relations, and functions, (2) presenting and analyzing mathematical problems, (3) structuring using algebraic symbols, (4) using mathematical models to present and understand quantitative relationships, and (5) analyze changes in various contextual problems [4]. The basic concepts of algebra need to be mastered to make it easier to learn more complex algebraic concepts [3]. To make it easier to learn algebraic concepts, students can minimize errors by mastering the prerequisite algebraic concepts [5]. Mastery of algebraic concepts is related to mastery of other mathematical concepts [6]. Algebraic concepts are often presented in the context of everyday problems, such as in finance and technology [7].

Students still have difficulty learning algebraic concepts [8]. Students still make mistakes so they are unable to solve algebra problems correctly. Errors that occur repeatedly can indicate the possibility that students have not mastered the concept of algebra [2]. Mistakes made by students are part of the student learning process [9]. Types of student difficulties can be seen from the errors that occur in the dimension of knowledge. The dimension of knowledge in education includes factual knowledge, conceptual knowledge, procedural knowledge, and metacognitive knowledge [7]. The basic difficulty experienced by students in learning algebra is the difficulty of using mathematical symbols in the form of letters and numbers simultaneously [6]. Students' learning difficulties are influenced by internal and external factors. Internal factors in the form of mathematical knowledge and abilities, motivation, anxiety, and mathematical beliefs. External factors include the influence of the social environment and learning styles [10].

Contextual problems are easier to solve by converting them into mathematical models, such as algebraic expressions, graphs, tables, and diagrams [8]. The process of making mathematical models of contextual problems is called the process of mathematics [11]. In algebra, mathematics means the process of changing contextual problems into algebraic symbols and vice versa [12]. Mathematics is divided into two categories, namely horizontal mathematics, and vertical mathematics. Horizontal mathematics means that contextual problems are broken down into language and mathematical symbols. Vertical mathematics means compiling procedures with various representations of mathematical models to solve mathematical problems [13]. The stages of the mathematical process are understanding the problem, formulating mathematical models, mathematical problem solving, and reflection [14].

Types of algebraic difficulties in the dimensions of factual knowledge such as difficulties in determining what is known and asked in an algebra problem [14]. In the conceptual knowledge dimension, algebraic difficulties are experienced by students such as difficulties in making representations of contextual problems into mathematical models [15]. Students' limited procedural knowledge makes students experience difficulties such as difficulty implementing arithmetic operations in algebraic form [16]. Students also experience difficulties with their limited metacognitive knowledge, such as difficulty making variations in the problem-solving process [7].

This study aims to determine the types of student difficulties in solving algebraic problems and to categorize these difficulties based on the mathematical stage. Each student has an algebraic problem-solving procedure that may be different. So that teachers need to master the concepts of algebra that are taught well. Teachers also need to understand the types of difficulties students experience in solving algebraic problems. That way, teachers can develop and develop appropriate learning strategies. So that it can reduce student difficulties and can develop students' mathematical abilities.

1. Methods

This study uses the systematic literature review method by taking the following steps: (1) formulating the research question, (2) developing the research protocol, (3) determining the source of the article data, (4) determining the criteria of the article, (5) evaluating the quality of the article, (6) extracting data from the selected article, (7) synthesizing the results, and (8) presenting the results [17].

## Research question

The research questions in this study are what types of difficulties students have in solving algebraic problems, and what are the types of algebraic difficulties that occur in each stage of mathematics.

## Data collection

This study synthesizes articles from previous research results as data sources. Search and selection of articles relevant to this study topic are in accordance with the following conditions.

* + 1. Database searched. The data source articles were obtained from various reputable scientific journals, such as: ERIC, Science Direct, DOAJ, and Sinta. Keywords that are used to find data source articles are "algebraic difficulties", "algebraic errors", and "mathematization".
		2. Selection criteria of papers. The data source articles were selected by taking into account the following criteria: (1) the topic of the article was relevant to the study of students' difficulties in mathematicalizing algebra problems, (2) the article was published in online journals in the period 2009-2020, and (3) the research subjects were students in education. formal from elementary school to college.

## Data analysis

The data source article is selected according to criteria 2.2.2 so that the data obtained can answer research questions.

* + 1. Extraction of selected papers. 17 articles that met the criteria were analyzed to obtain data about authors, published dates, journal publishers, research methods, research subjects, subject locations, and research results.
		2. Synthesis of results. Synthesis of research data is prepared based on data extraction in 2.3.1. The data shows the types of students' difficulties in solving algebraic problems observed from the dimensions of factual, conceptual, procedural, and metacognitive knowledge. The data also displays difficulty classifications based on the mathematical stage.
1. Result and discussion

Table 1 shows the results of the search for articles that match 2.2.1 and meet the criteria in 2.2.2.

**Table 1.** Result data of selected article extraction.

| **No** | **Author (Years)** | **Journal** | **Research method** | **Subject** | **Country** |
| --- | --- | --- | --- | --- | --- |
| 1 | L Sugiarti and H Retnawati (2019) [16] | ISIMMED2018 | Descriptive qualitative | Junior High School | Indonesia |
| 2 | Ralivia Suci Setianingrum, Syamsuri, Yani Setiani (2020) [7] | MaPan : Jurnal Matematika dan Pembelajaran | Descriptive qualitative | Junior High School | Indonesia |
| 3 | Ekawati (2018) [10] | A Journal of Language, Literature, Culture, and Education POLYGLOT | Descriptive qualitative | Junior High School | Indonesia |
| 4 | Craig Pournara, Jeremy Hodgen, Yvonne Sanders, Jill Adler (2016) [18] | Pythagoras - Journal of the Association for Mathematics Education of South Africa | Mix method (Quantitative-Qualitative) | Secondary School | South Africa |
| 5 | Al Jupri, Paul Drijvers (2016) [14] | Eurasia Journal of Mathematics, Science & Technology Education | Design research | Secondary School | Indonesia |
| 6 | Mary Mulungye M., Dr. Miheso O‘Connor, Dr. Ndethiu S. (2016) [19] | Journal of Education and Practice | Descriptive survey design | Secondary School | Kenya |
| 7 | Julie L. Booth, Christina Barbieri, Francie Eyer, and E. Juliana Paré-Blagoev (2014) [20] | Journal od Problem Solving | Descriptive Quantitative | Secondary school | United States |
| 8 | A Jupri, D Usdiyana and R Sispiyati (2019) [12] | MSCEIS 2018 | Design research | Junior High School | Indonesia |
| 9 | Al Jupri, Paul Drijvers, Marja van den Heuvel-Panhuizen (2014) [8] | Math Ed Res J Mathematics Education Research Group of Australasia | Design research | Junior High School | Indonesia |
| 10 | Gunawardena Egodawatte (2009) [15] | Acta Didactica Napocensia | Mix method (Quantitative-Qualitative) | Secondary school | Sri Lanka |
| 11 | Joan Lucariello, Michele T. Tine, Colleen M.Ganley (2013) [21] | Journal of Mathematical Behavior | Quantitative research | Secondary school | United States |
| 12 | Ömer Sahin, Yasin Soylu (2011) [1] | Procedia Social and Behavioral Sciences | Descriptive qualitative | Elementary school | Turkey |
| 13 | Indah Puspitasari Maharani, Subanji Subanji (2018) [6] | International Electronic Journal Of Mathematics Education | Mix method (Quantitative-Qualitative) | Junior High School | Indonesia |
| 14 | Gözde Akyüz (2015) [2] | Community College Journal of Research and Practice | Mix method (Quantitative-Qualitative) | Vocational school | Turkey |
| 15 | Marta Molina, Susana Rodríguez-Domingo, María Consuelo Cañadas, Encarnación Castro (2016) [22] | Int J of Sci and Math Educ | Design research | Secondary school | Spain |
| 16 | Effandi Zakaria, Ibrahim, Siti Mistima Maat (2010) [3] | International Education Studies | Survey | Secondary school | Indonesia |
| 17 | Judah P. Makonye, Duduzile Winnie Khanyile (2015) [9] | Research in Education | Qualitative research | Secondary school | South Africa |

Articles that meet the 2.2.2 criterion are extracted according to 2.3.1. The data from the extraction of articles were analyzed and synthesized to determine the types of student difficulties in solving algebraic problems observed from the dimensions of factual, conceptual, procedural, and metacognitive knowledge. The synthesized data also displays the classification of types of difficulties based on the stage of the mathematical process. The stage of mathematicalization is understanding the problem, compiling a mathematical model, solving mathematical problems, and reflecting. Table 2 shows the results of the synthesis of the types of student difficulties in solving algebraic problems.

**Table 2.** Results of the synthesis of difficulties solving algebra problems.

|  |  |  |
| --- | --- | --- |
| **Dimensions of knowledge** | **Difficulty in algebra** | **Number of research\*** |
| Factual knowledge | Understanding sentences | 2 |
| Ignoring facts | 1 |
| Conceptual knowledge | Changing problems to algebraic expressions | 6 |
| Making mathematical representation | 3 |
| Procedural knowledge | Applying variabel concepts | 7 |
| Applying arithmetical operations | 9 |
| Using algebraic expressions | 8 |
| Solving equality and inequality | 5 |
| Simplifying algebraic fractions | 4 |
| Factoring algebraic expressions | 5 |
| Understanding bracket usage | 4 |
| Metacognitive knowledge | Checking procedure | 2 |
| Resolving problems with various process | 1 |

\*some articles examine more than one difficulties

Table 2 shows the types of student difficulties in solving algebraic problems which will be grouped into phase of mathematization.

## Understanding the problem

Understanding sentences and ignoring facts are a type of algebraic difficulty in the dimensions of factual knowledge. Both types of difficulties are included in the mathematical stage of understanding the problem. Students who are faced with contextual problems, for example in the form of story problems, tend to have difficulty identifying what facts are known and what are being asked [10]. Inappropriate understanding of the problem results in students making mistakes in solving the problem. The information in the questions is not all important, sometimes there is confusing information. Students who ignore important information in the questions tend to make the process of solving and answering the wrong things [14].

## Formulating mathematical models

The types of algebraic difficulties at the stage of compiling a mathematical model are seen from errors in the dimensions of conceptual knowledge, namely changing problems to algebraic expressions and making mathematical representation. Algebraic contextual problems can be solved by converting them into algebraic expressions [16]. Students will find it difficult if they do not understand the concept of determining and composing algebraic expressions [10]. During the problem solving process, various representations of mathematical models are needed. Such as algebraic expressions that need to be converted into tables or diagrams. So that students need to understand how to arrange a problem into various representations of mathematical models [15].

## Mathematical problem solving

The types of difficulties in the procedural knowledge dimension can be categorized in the mathematical process stage of solving mathematical problems, namely applying variable concepts and arithmetical operations, using and factoring algebraic expressions, solving equality and inequality, simplifying algebraic fractions, and understanding bracket usage.

* + 1. Applying variabel concepts. Students must master the concept of variables well. Students have difficulty arranging variables as labels of a fact in the problem. Students consider a variable to have a specific value. Students have difficulty determining the value of a variable. Another student error is that the student eliminates variables when performing arithmetic operations [1], [14].
		2. Applying arithmetical operations. Students have difficulty applying arithmetic operations in algebraic expressions, such as addition, subtraction, multiplication, and division operations in algebraic expressions. Students have difficulty applying commutative, associative, distributive, and inverse properties in algebraic expressions [8]. Operating negative signs in algebraic expressions also makes students difficult. Students have difficulty distinguishing between negative numbers and subtraction hands [2].
		3. Using and factoring algebraic expressions. The mistake made by students is that students assume that algebraic expressions always have integer results. The concept of duality in algebraic expressions is shown in the addition of algebraic expressions. Addition of algebraic expressions is considered to be addition, but also as an algebraic object [14]. Students factor each term in an algebraic expression [2].
		4. Solving equality and inequality. Students need to understand the meaning of the equal sign "=" in algebraic expressions. The equal sign "=" in algebra means "is algebraically equivalent to". This is different from the meaning of the equal sign in number operations which means "result of" [8]. Students have difficulty applying the inequality symbol less than "<" and more than ">" [10].
		5. Simplifying algebraic fractions. The concept of operating rules in algebraic fractions needs to be well understood by students [6]. Students experience errors in applying the operations of addition, subtraction, multiplication, and division of algebraic fractions. Students who ignore the rules for operating fractions will have difficulty simplifying and solving fractions algebraically [2]. Errors made by students in simplifying algebraic fractions include cancellation error, confusing the factors, no recognition of the common factor, unable to factorise a trinomial, and dropping the denominator [9].
		6. Understanding bracket usage. Students must understand the concept of using parentheses in algebraic expressions. Ignoring parentheses when solving algebraic equations can result in wrong answers [1], [15]. Students have difficulty interpreting the power of bracket. This happens because of a misunderstanding of the application of the distributive nature [19].

## Reflection

Types of algebraic difficulties at the reflection stage are seen from errors in the dimensions of metacognitive knowledge, namely checking procedures and resolving problems with various processes. Many students experience calculation errors and mistakes in choosing the completion procedure. This is because students do not check the problem-solving process again [7]. Students will experience difficulties when they have solved a problem with a certain procedure, then asked to complete it again in a different way. Good conceptual understanding and creative thinking skills are needed in order to devise several solving procedures for a problem [14].

Based on the explanation of the results and discussion above, the types of student difficulties in solving algebraic problems can be classified into the stages of the mathematical process as shown in Figure 1.

Procedural knowledge

Metacognitif knowledge

Conceptual knowledge

Factual knowledge

Understanding sentences

Ignoring facts

Checking procedure

Understanding bracket usage

Changing problems to algebraic expressions

Factoring algebraic expressions

Simplifying algebraic fractions

Making mathematical representation

Applying variabel concepts

Applying arithmetical operations

Using algebraic expressions

Solving equality and inequality

Resolving problems with various process

Understanding the problem

Formulating mathematical models

Mathematical problem solving

Reflection

**Mathematizing**

**Figure 1.** Types of student difficulties in mathematicalizing algebra problems.

1. Conclusion

Types of student difficulties in solving algebraic problems can be observed from errors in the dimensions of factual, conceptual, procedural, and metacognitive knowledge. These types of difficulties are categorized based on the stages of the mathematical process. Teachers are expected to be able to understand students 'difficulties in solving algebraic problems so that teachers can develop appropriate learning strategies to develop students' mathematical abilities. In this study, it was found that at the stage of understanding the problem, the difficulties that occurred were difficulty understanding the sentence of the problem and ignoring the facts of the problem. The difficulty in compiling a mathematical model is compiling problems into algebraic expressions and compiling various mathematical representations. The stage of solving math problems covers most of the difficulties, namely applying the concept of variables, applying arithmetic operations, using algebraic expressions, solving equations and inequalities, simplifying algebraic fractions, factoring algebraic forms, and using parentheses. Then the difficulties that occur in the reflection stage are evaluating problem-solving procedures and solving problems in other ways.

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