Analysis of student conception on exponent material using three tier test

S Fadillah1, D Oktaviana2 and R Astuti3

1,2,3Departement of Mathematics Education, IKIP PGRI Pontianak, Indonesia

e-mail: syffadillah@gmail.com

**Abstract**. This study aimed to (1) describe the students' conception of class X in exponent material and (2) describe the misconceptions experienced by students of class X in exponent material. Subjects in this study were the students of class X at one of vocational school in West Kalimantan. The instrument used in this study was a three-tier test, a test consisting of three levels, the first level is a multiple choice question regarding the concept of exponents, the second level is a matter of reasoning about the reason for the answer from the first step, and the third level questions about confidence in the answers given by students. The results of this study indicate that 69.71% of students understand the concept, 14.71% of students experience misconceptions, 0.59% of students experience misconceptions (false positive) and 0.88% experience misconceptions (false negative), 7.65% of students do not understand the concept, and 6.45% of students are identified correctly because of luck or true but not confident. Misconceptions experienced by students in learning exponent material are students do not understand the properties of exponents so students are wrong in using exponential properties to simplify exponential equations involving the operation of addition, subtraction, multiplication, division, and rank. Misconceptions that also occur a lot are students wrong in understanding the definition of negative exponent.

1. Introduction

Mathematics is a subject full of concepts. If one concept is not understood, it will affect the understanding of other concepts because the concepts are interrelated. That is, an understanding of basic concepts is needed so that later it will be easier to understand the following concepts. To understand students' conceptions, several different types of instruments are used to indicate them, such as interviews, open questions, concept maps, and multiple-choice questions all of which have advantages and disadvantages in their use practices [1]. Multiple-choice tests are often preferred because it is easy to apply in measuring students' understanding of a concept/ material, but regular multiple-choice tests have some limitations in their application, such as in determining whether students respond correctly to tests consciously or accidentally. On the other hand, interviews can provide more complete information about alternative concepts of students and their understanding related to certain concepts, but it takes a long time to carry out interviews with many students and generalize their concepts.

The three-tier test is one instrument that can be used to identify students' conceptions. This instrument proved to be very accurate in observing the effectiveness of learning, measuring student understanding, and distinguishing students' misconceptions due to not understanding concepts or misconceptions [2]. Three-tier test is an instrument with three levels, namely: (1) content tier that measures respondents' knowledge related to a concept/material, (2) reason tier to see the reasons behind the answers given by respondents in the content tier, and (3) certainty response index which measures how confident the respondent will be in the first and second level [3]. This type of test is considered to be able to diagnose students 'conceptions/misconceptions well because there is a second level of test that asks the reasons for the respondents' answers at the first level. And also the third level that asks how confident respondents are with the answer ([1], [2], [3], [4]).

When students learn about a concept, in general, there are three possibilities, namely: students understand the concept, students do not understand the concept, and students experience misconceptions in understanding the concept. Three-tier tests can further identify whether the truth of students in answering questions because students understand the concept or correct only because of luck (students lack confidence in answering questions). Likewise, when students answer incorrectly, a three-tier test will be able to identify whether the error is because students do not understand the concept or students experience misconceptions in understanding the concept.

When teachers teach a concept to students, teachers need to know the students 'conceptions so that they can evaluate students' understanding of concepts and evaluate how to teach them. When students do not understand the concept, the teacher can re-teach to give students an understanding of the concept. When students experience misconceptions, the teacher must explain students' misconceptions in understanding the concepts, and explain the correct concepts. Misconceptions will interrupt the acquisition of new knowledge because misconceptions in understanding the prerequisite concept are believed to hold back the new knowledge acquisition and integration [5].

Exponent is one of the mathematical materials learned in vocational high schools in class X. exponentiation is important for the understanding of advanced mathematical concepts, such as exponential functions, logarithm, and calculus. Furthermore, exponentiation is crucial to interpret some of the models in different fields, such as population control, radioactive decay, problems of inflation, musical scales, algebra, and complex analysis [6]. The difficulty of students in the exponent concept will cause students to also experience difficulties in learning advanced mathematical concepts. Based on the background that has been mentioned, the writer feels interested in doing research related to the conception of vocational students on exponent material. This study aims to describe the students' conceptions and misconceptions experienced by students of class X in exponent material

1. Method

The method used in this research is descriptive because this research will describe students' conceptions and misconceptions in exponent material. The research subjects were grade X vocational school students in one school in West Kalimantan who had studied exponent material. The instrument used was a three-tier test, a test with three levels consisting of 10 questions. The indicators of the test are presented in Table 1.

**Table 1.** Indicators of Exponent Test Questions

|  |  |  |
| --- | --- | --- |
|  | Indicators | Question Number |
| **1** | Multiplication of exponential expressions with the same bases. | 1, 2 |
| **2** | Multiplication of exponential expressions with different bases. | 7 |
| **3** | Division of exponential expressions with the same bases. | 3, 5 |
| **4** | Division of exponential expressions with different bases. | 4 |
| **5** | Rank of exponential expressions | 6, 8 |
| **6** | Division and rank of exponential expressions with different bases. | 9 |
| **7** | Addition of negative exponent with the same bases. | 10 |

Three-tier diagnostic test consists of three levels, the first level is a multiple choice question with five answer choices (A, B, C, D, and E) regarding the concept of exponents, the second level is a matter of reasoning about the reason for the answer from the first step, this reason also in the form of multiple choices consisting of five answer choices (A, B, C, D and E). The third level is the certainty of response index of questions about confidence in the answers given by students, consisting of two choices, namely sure and not sure.

The procedures in this study are: (1) giving three-tier tests to research subjects, (2) grouping students’ conceptions based on students' answers to the three-tier tests, (3) calculating the percentage of students’ conceptions, (4) describing students' conceptions, especially on concepts that are most students experience the most misconceptions. The grouping of students' conceptions based on answers to the three-tier tests given by respondents can be seen in the following Table 2.

**Table 2.** Student Conception Based on Answers on the Three-tier Test

|  |  |
| --- | --- |
| Student Answers on the Three-tier Test | Student Conception |
| Level 1 | Level 2 | Level 3 |
| correct | correct | sure  | understand |
| correct | incorrect | sure | misconception (false positive) |
| incorrect | correct | sure | misconception (false negative) |
| incorrect | incorrect | sure | misconception |
| correct | correct | unsure | lucky guess, lack of confidence |
| correct | incorrect | unsure | do not understand |
| incorrect | correct | unsure | do not understand |
| incorrect | incorrect | unsure | do not understand |

1. Results and discussion

Based on students' answers on the three-tier diagnostic test, students are grouped into five categories, namely understanding concepts (U), misconceptions (M), false-positive misconceptions (MFP), false-negative misconceptions (MFN), not understanding concepts (NU), and true because of lucky guess or true but not confident (LG). Students' conceptions on each item from the three-tier test are presented in Table 3.

Table 3. Students' Conceptions on Each Item from the Three-Tier Test

|  |  |
| --- | --- |
| Number of Question | Percentage |
| U | M | MFP | MFN | DU | LG |
| 1 | 85.30% | 0% | 2.94% | 2.94% | 8.82% | 0% |
| 2 | 94.12% | 2.94% | 0% | 2.94% | 0% | 0% |
| 3 | 82.35% | 2.94% | 2.94% | 0% | 11.77% | 0% |
| 4 | 55.89% | 26.47% | 0% | 0% | 8.82% | 8.82% |
| 5 | 64.70% | 17.65% | 0% | 0% | 5.88% | 11.77% |
| 6 | 76.47% | 8.82% | 0% | 0% | 2.94% | 11.77% |
| 7 | 82.35% | 8.82% | 0% | 0% | 2.94% | 5.89% |
| 8 | 85.30% | 5.88% | 0% | 0% | 2.94% | 5.88% |
| 9 | 29.41% | 52.94% | 0% | 0% | 14.71% | 2.94% |
| 10 | 41.17% | 20.59% | 0% | 2.94% | 17.65% | 17.65% |
| Average | 69.71% | 14,71% | 0.59% | 0.88% | 7.65% | 6.47% |

Based on Table 3, it was concluded that mastery of students' concepts on exponents was quite good. This can be seen from the average of students who understand the concept of 69.71%. But there are still 16.18% of students experiencing misconceptions (14.71% misconceptions, 0.59% false-positive misconceptions, and 0.88% false-negative misconceptions) and 7.65% of students do not understand the concept of exponents. Based on Table 3, it is also seen that many students experience mistakes in questions number 4, 5, 9, and 10, which is less than 65% of students who understand the concepts in these questions. The following will describe the mistakes made by students in solving questions number 4, 5, 9, and 10. Question number 4 of the three-tier test as in Table 4.

**Table 4.** Three-tier Test: Question Number 4

|  |  |
| --- | --- |
| Tiers | Three-Tier Question |
| Tier 1 | 4. 1. Simplify the exponents: $\frac{x^{6} y^{6} z^{6}}{x^{-2} y^{-3} z^{-4}}= . . . . $1. $x^{4} y^{3} z^{2}$
2. $x^{8} y^{9} z^{10}$
3. $x^{-3} y^{-2}z^{-\frac{3}{2}}$
4. $\left(xyz\right)^{27}$
5. $x^{-12} y^{-18} z^{-24}$
 |
| Tier 2 | * 1. Which one of the followings is the reason of your answer for the previous question?
1. $\frac{x^{6} y^{6} z^{6}}{x^{-2} y^{-3} z^{-4}}=x^{6-2}y^{6-3}z^{6-4}$
2. $\frac{x^{6} y^{6} z^{6}}{x^{-2} y^{-3} z^{-4}}=x^{6-(-2)}y^{6-(-3)}z^{6-(-4)}$
3. $\frac{x^{6} y^{6} z^{6}}{x^{-2} y^{-3} z^{-4}}=x^{6×(-2)}y^{6×(-3)}z^{6×(-4)}$
4. $\frac{x^{6} y^{6} z^{6}}{x^{-2} y^{-3} z^{-4}}=x^{6÷(-2)}y^{6÷(-3)}z^{6÷(-4)}$
5. $\frac{x^{6} y^{6} z^{6}}{x^{-2} y^{-3} z^{-4}}=\left(xyz\right)^{6+6+6}-\left(xyz\right)^{-2+\left(-3\right)+\left(-4\right)}$ $=\left(xyz\right)^{18}-\left(xyz\right)^{-9}=\left(xyz\right)^{18-(-9)}$
 |

In question number 4, 26.47% of students experienced misconceptions and 8.82% of students do not understand the concept. Misconceptions that occur in this problem are students answering at tier 1 is E and tier 2 is C ($\frac{x^{6} y^{6} z^{6}}{x^{-2} y^{-3} z^{-4}}=x^{6×(-2)}y^{6×(-3)}z^{6×(-4)}=x^{-12} y^{-18} z^{-24})$. This misconception occurs because students incorrectly apply the properties of exponent, students simplify the division of exponential expressions by using multiplication properties, so students add the power of exponential expressions, not subtraction it. Another misconceptions are students answering at tier 1 is A and tier 2 is A $(\frac{x^{6} y^{6} z^{6}}{x^{-2} y^{-3} z^{-4}}=x^{6-2}y^{6-3}z^{6-4}=x^{4} y^{3} z^{2})$. This misconception occurs because students do not pay attention to the negative exponent of the divisor. Question number 5 of the three-tier test as in Table 5.

**Table 5.** Three-tier Test: Question Number 5

|  |  |
| --- | --- |
| Tiers | Three Tier Question |
| Tier 1 | * 1. Calculate the operating results of the exponents: $\frac{2^{4}}{2^{2}}+\frac{3^{4}}{3^{2}}= . . . . .$
1. 625
2. 25
3. $\frac{97}{13}$
4. 793
5. 13
 |
| Tier 2 | * 1. Which one of the following is the reason for your answer to the previous question?
1. $\frac{2^{4}}{2^{2}}+\frac{3^{4}}{3^{2}}= \frac{5^{4}}{5^{2}}=\frac{625}{25}$
2. $\frac{2^{4}}{2^{2}}+\frac{3^{4}}{3^{2}}= \frac{5^{8}}{5^{4}}=5^{8-4}=5^{4}$
3. $\frac{2^{4}}{2^{2}}+\frac{3^{4}}{3^{2}}=2^{4+2}+3^{4+2}=2^{6}+3^{6}$
4. $\frac{2^{4}}{2^{2}}+\frac{3^{4}}{3^{2}}=\frac{2^{4}+3^{4}}{2^{2}+3^{2}}=\frac{16+81}{4+9}$
5. $\frac{2^{4}}{2^{2}}+\frac{3^{4}}{3^{2}}=2^{4-2}+3^{4-2}=2^{2}+3^{2}$
 |
| Tier 3 | 5.3. Are you sure about your answers for the previous two questions;1. I am sure.
2. I am not sure.
 |

In question number 5, 17.65% of students experienced misconceptions and 5.88% of students do not understand the concept. The mistake made by students who are misconceptions and students who do not understand the concept is that the answer to tier 1 is B and tier 2 is A $(\frac{2^{4}}{2^{2}}+\frac{3^{4}}{3^{2}}= \frac{5^{4}}{5^{2}}=\frac{625}{25}=25)$. This shows that students experience misconceptions in the operation of addition and division of exponent. A similar mistake was also found by [7] in his research which concluded that one of the difficulties of students in exponential material is in carrying out the operation of addition and multiplication of exponents.

In this question, enough students answered correctly in tier 1 and tier 2, but they not sure that their answers were correct. This likely happened because students guessed when answering questions. However, this can also be caused the students do not understand the concept so that they lack confidence in answering questions. Question number 9 of the three-tier test as in Table 6.

**Table 6.** Three-tier Test: Question Number 9

|  |  |
| --- | --- |
| Tiers | Three Tier Question |
| Tier 1 | 9.1. Simplify the exponents: $\left(4a^{3}\right)^{2} :2a^{4}= . . . . .$1. $8a^{-1}$
2. $2a^{3}$
3. $8a^{2}$
4. $2a^{1}$
5. $2a^{2}$
 |
| Tier 2 | * 1. Which one of the followings is the reason of your answer for the previous question?
1. $\left(4a^{3}\right)^{2} :2a^{4}=\frac{4^{2}\left(a^{3}\right)^{2}}{2a^{4}}=\frac{16a^{6}}{2a^{4}}=8a^{6-4}$
2. $\left(4a^{3}\right)^{2} :2a^{4 }= \frac{4a^{6}}{2a^{4}}=2a^{6-4}$
3. $\left(4a^{3}\right)^{2} :2a^{4}= \frac{4a^{5}}{2a^{4}}=2a^{5-4} $
4. $\left(4a^{3}\right)^{2} :2a^{4}=\frac{4^{2}a^{3}}{2a^{4}}=\frac{16a^{3}}{2a^{4}}=8a^{3-4} $
5. $\left(4a^{3}\right)^{2} :2a^{4}=\frac{4a^{6}}{2a^{4}}=2a^{6÷2}=2a^{3} $
 |
| Tier 3 | 9.3. Are you sure about your answers for the previous two questions;1. I am sure.
2. I am not sure.
 |

Question number 9 is the question with the lowest score, only 29,41% of students understand the concept, 52,94% of students experienced misconceptions, 14.71% of students do not understand the concept. This is the most misconception done by students. Most students experience the same misconception, namely answering E at tier 1 and B at tier 2 ($\left(4a^{3}\right)^{2} :2a^{4 }= \frac{4a^{6}}{2a^{4}}=2a^{6-4}$). Another misconception is that students answer D in tier 1 and C in tier 2 $(\left(4a^{3}\right)^{2} :2a^{4}= \frac{4a^{5}}{2a^{4}}=2a^{5-4})$. This misconception occurs because students do not understand the properties of the powers of exponents. Question number 10 of the three-tier test as in Table 7.

**Table 10.** Three-tier Test: Question Number 7

|  |  |
| --- | --- |
| Tiers | Three Tier Question |
| Tier 1 | 10.1. Calculate the operating results of the exponents: $4^{-2}+4^{-3}-4^{-1}= . . . . . $1. $-\frac{1}{^{1}/\_{4096}}$
2. $\frac{11}{64}$
3. $\frac{1}{^{1}/\_{4096}}$
4. $\frac{1}{4096}$
5. $-\frac{11}{64}$
 |
| Tier 2 | * 1. Which one of the followings is the reason of your answer for the previous question?
1. $4^{-2}+4^{-3}-4^{-1}=\frac{1}{4^{2}}+\frac{1}{4^{3}}-\frac{1}{4^{1}}=\frac{1}{16}+\frac{1}{64}-\frac{1}{4}=\frac{4+1-16}{64}=-\frac{11}{64}$
2. $4^{-2}+4^{-3}-4^{-1}=\left(4\right)^{-2+\left(-3\right)+(-1)}=4^{(-6)}=\frac{1}{4096} $
3. $4^{-2}+4^{-3}-4^{-1}=\frac{1}{\left(4\right)^{-2-3-1}}=\frac{1}{4^{-6}}=\frac{1}{^{1}/\_{4096}}$
4. $4^{-2}+4^{-3}-4^{-1}=\frac{1}{\left(4+4-4\right)^{-2+\left(-3\right)+(-1)}}=\frac{1}{4^{-6}}=-\frac{1}{^{1}/\_{4096}}$
5. $4^{-2}+4^{-3}-4^{-1}=\frac{1}{-4^{2}}+\frac{1}{-4^{3}}-\frac{1}{-4^{1}}=-\frac{1}{16}-\frac{1}{64}+\frac{1}{4}=\frac{-4-1+16}{64}=\frac{11}{64}$
 |
| Tier 3 | * 1. Are you sure about your answers for the previous two questions;
1. I am sure.
2. I am not sure.
 |

In question number 10, 20.59% of students experienced misconceptions, 17.65% of students do not understand the concept. Misconceptions that occur in this problem are students answering at tier 1 is B and tier 2 is E$ (4^{-2}+4^{-3}-4^{-1}=\frac{1}{-4^{2}}+\frac{1}{-4^{3}}-\frac{1}{-4^{1}}=-\frac{1}{16}-\frac{1}{64}+\frac{1}{4}=\frac{-4-1+16}{64}=\frac{11}{64})$. This misconception occurs because students misunderstand the definition of negative exponents. Another misconceptions are students answering at tier 1 is D and tier 2 is B ($4^{-2}+4^{-3}-4^{-1}=\left(4\right)^{-2+\left(-3\right)+(-1)}=4^{(-6)}=\frac{1}{4096})$. This misconception occurs because students incorrectly apply the properties of exponent, students simplify the addition of exponential expressions by using multiplication properties. A similar mistake was discovered by [8] who in her research found that students made many mistakes in understanding the concept of negative exponents.

In general misconceptions experienced by students in learning exponent material are students do not understand the properties of exponents so students are wrong in using exponential properties to simplify exponential equations involving the operation of addition, subtraction, multiplication, division, and rank (e.g. $\frac{x^{6}}{x^{-2}}= x^{6-2}; \frac{x^{6}}{x^{-2}}= x^{6×(-2)}; \frac{2^{4}}{2^{2}}+\frac{3^{4}}{3^{2}}= \frac{5^{4}}{5^{2}}; 4^{-2}+4^{-3}= 4^{-2+(-3)};\left(4a^{3}\right)^{2}=4a^{5};\left(4a^{3}\right)^{2}=4a^{6}$. Misconceptions that also occur a lot are students wrong in understanding the definition of negative exponents (e.g. $4^{-2}=\frac{1}{-4^{2}})$.

Students' misconceptions in learning exponential material are students’ learning obstacles. The misconception analysis produced in this study can be the basis for developing learning designs through didactical design research (DDR). Some researchers have used students’ learning obstacles in designing mathematics learning designs through DDR successfully overcoming students’ learning obstacles in learning mathematics ([9], [10], [11]).

1. Conclusions

Based on the results of the study concluded that 69.71% of students understand the concept, 16.18% of students experiencing misconceptions (14.71% misconceptions, 0.59% false-positive misconceptions, and 0.88% false-negative misconceptions), 7.65% of students do not understand the concept, and 6.45% of students are identified correctly because of luck or true but not confident. Some concepts that students have mastered well the multiplication of exponents with the same and different bases, the division of exponents with the same and different bases, and the exponent's rank. But many students experience misconceptions in problems with varying operations, students incorrectly use exponential properties to simplify exponential equations that involve addition, subtraction, multiplication, division, and rank operations. Misconceptions are also experienced by students on negative exponents.

1. Acknowledgments

The author would like to express her gratitude to the teachers and students in SMKN 4 in Pontianak that made this research possible.

References

1. Cetin A, Dindar and Geba O 2011 Development of a three-tier test to assess high school students understanding of acids and basses *Procedia Social and Behavioral Sciences* **15** 600-4
2. Pesman H and Eryilmaz A 2010 Development of a three-tier test to assess misconceptions about simple electric circuits *The Journal of Educational Research* **103** 208-22
3. Arslan HO, Cigdemoglu C and Moseley C 2012 A three-tier diagnostic test to assess pre-service teachers’ misconceptions about global warming, greenhouse effect, ozone layer depletion, and acid rain. *International Journal of Science Education* **34** 1667-86
4. Caleon I and Subramaniam R 2010 development and application of a three-tier diagnostic test to assess secondary students’ understanding of waves. *International Journal of Science Efucation,* **32** 939-61
5. Gollub JP, Bertenthal MW, Labov JB and Curtis PC, *Learning and**Understanding Improving Advanced Study of Mathematics and Science In U.S. High Schools* 2002 (Washington:National Academy Press)
6. Ellis AB, Ozgur Z, Kulow T, Williams CC and Amidon J 2015 Quantifying exponential growth: three conceptual shifts in coordinating multiplicative and additive growth *The Journal of Mathematical Behavior* **39** 135-55
7. Ulusoy F 2019 Serious obstacles hindering middle school students’ understanding of integer exponents *International Journal of Research in Education and Science.* **5** 52-69
8. Nuriyah FE 2014 The result of assessment for students in solving exponents and logarithms problems *Proceeding of International Conference on Educational Research and Evaluation (ICERE)**Yogyakarta State University, Indonesia*
9. Nur’aeni HE and Muharram MRJ 2016 Didactical design research of mathematical communication about concept of cuboid volume in elementary school”. *Proceeding of 3RD International Conference On Research, Implementation and Education of Mathematics and Science Yogyakarta, Indonesia*
10. Sulisti H, Sugiatno and Sayu S 2018 Students’ mathematical communication ability and disposition in the implementation of micro didactic design at junior high school. *Journal of Education, Teaching and Learning* **3** 379-86
11. Alawiyah A, Waluya SB and Prasetyo APB 2018 Didactical situations of students’ mathematical reasoning based on the learning obstacle on quadrilateral areas *Unnes Journal of Mathematics Education Research* **7** 196-203