Students’ Mathematical Representation in Solving Mathematical Problems

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**Abstract**. Mathematical representation is an ability to process and reveal mathematical ideas/concepts to discover the solution to a problem by changing it to another form. Mathematical representation is one of the essential skills of students. When students build ideas and use mathematical representation, they have developed basic skills indirectly that help them in solving problems. There are various types of mathematical representations to solve mathematical problems. In the in-class mathematics learning, a representation should not only be related to change one form to another in one way, but it can be done in two ways or more. Therefore, students should be provided with problems in which their solutions can be achieved using various types of representations. This article discusses mathematical representation types to be used in solving mathematical problems, i.e., verbal, visual, and symbolic representations.

1. Introduction

Mathematics is a science to solve problems in daily life and as the base to understand other sciences. Learning mathematics trains us to think logically, critically, and creatively so that students are used to facing and solving problems. It demands mathematical representation ability where students can illustrate, explain, or expand their mathematical ideas by focusing on its essential forms [1].

In the in-class learning, students will face several mathematical problems. They are going to try to understand the problem and finish it with their known formulas. These formulas are incredibly related to previous knowledge that corresponds to the given problem. The model or the representation to be made may vary according to each individual's ability to interpret the existing problem [2].

Representation was also included in the five mathematics abilities implemented by the NCTM (National Council of Teacher of Mathematics). NCTM [3] determined the representation standard to be mastered in the learning process, which can choose, implement, and conduct translation between mathematical representations in solving problems.

Students should understand that mathematical representation is a vital part of mathematics learning to facilitate mathematics learning and communicate with others regarding mathematics ideas. This representation ability will continue to develop according to age development or thought level development [4]. When students develop ideas and use mathematical representation, they unconsciously have developed the basic knowledge that helps them solve problems [5]. When students learn mathematics more-in-depth, they can extend their mathematics idea understanding or the relation by moving from one type of representation to another representation from the same relationship. The utilization of various representations is crucial, especially in understanding and interpreting mathematical concepts.

Mathematical representation is expected to help and ease students in solving mathematical problems. However, many students are still struggling to represent mathematical problems. As in the study of Rahmawati et al. [6], students experienced difficulties in determining the suitable symbolic representation with a relationship represented in graphs or tables. In another study conducted by Safiyah and Retnawati [7] stated that several difficulties were found in using illustrations to represent mathematics symbols, using mathematics symbols to represent mathematics symbols, using mathematics symbols to represent illustrations, using illustrations to represent written text statements, and using mathematics symbols to represent written text statements.

By realizing that mathematical representation is an essential aspect of mathematics learning and realizing that there are plenty of students who struggle to represent mathematical problems, teachers need to understand various types of mathematical representations and provide supporting learning to improve students' mathematical representation ability. This article discusses the definition and types of mathematical representations to be used in solving mathematical problems.

1. Mathematical Representation

Representation refers to the activity to process or produce; in other words, taking action to catch a mathematics concept by changing it to another form [3]. Representation also said to be the expression of mathematics ideas presented by students as a model transformation that changed from a problem to create a solution [8]. Representations that should be brought up by students are expressions of ideas or mathematics ideas presented by students to find the solution for the problem [9].

According to NCTM [3], there are three mathematical representation standards to be mastered by students in mathematics learning: 1) Create and use representation to organize, record, and communicate mathematics ideations. 2) Choose, use, and understand various mathematical representations to solve mathematical problems. 3) Use representation to model and understand physical, social, and mathematical phenomena.

According to Pape & Tchoshanov [10], there are four ideas used in understanding the representation concept, i.e., a representation may be viewed as an internal abstraction of mathematics ideas or cognitive schemata built by students through experiences, as a mental reproduction from the previous mental condition, as structured presentation through illustrations, symbols, or emblems, and as knowledge of something that represents another thing.

Representation defined as the configuration of characters, illustrations, concrete objects, and more that represents other things. Many experts divide mathematical representations into various types, such as graphs, illustrations, diagrams, verbal, manipulative forms, and symbols. Furthermore, representation also seen as a way or form in which students explain, symbolize, and relate to the mathematics concept [11].

In the in-class mathematics learning, a representation should not only be related to change one form to another in one way, but also in two ways or more [9]. For example, when the solution of a mathematics problem is presented in the graph representation form, teachers may ask students to create other representations like presenting it in the table, mathematics equation/model, or words.

Representation is a process to develop the mental possessed by someone revealed and visualized in various mathematical models such as verbal, illustrations, concrete objects, tables, manipulative models, or combinations of them [2]. According to Graciella and Suwangsih [8], there are three mathematical representations in mathematics learning to be used, i.e., visual, verbal, and symbolic. It follows a study of Bruner [12] that classified representations to be enactive, iconic, and symbolic.

*2.1. Visual Representation*

Visual representation is a form of representation that involves visualized graphs and diagrams [8]. A visual representation, such as illustration, is a relatively accurate description of someone’s emotional condition and is better than verbal representation [13]. In other words, it can be said that illustrations are equally accurate equipment to discover students’ affective experience.

* 1. *Verbal Representation*

Verbal representation is someone’s ability to adjust natural language on word, phrase, and sentence levels [8]. Verbal representation needs written language utilization to understand, illustrate, analyze, explain, or reflect numerical, algebra, or graph representations that do not include a brief phrase such as a guide to solving problems [14]. Verbal representation is commonly used to interpret the end result of a problem’s solving [15]. It can be said that verbal representation is a representation in the form of words/sentences to interpret mathematical problems solving.

* 1. *Symbolic Representation*

Symbolic representation is an ability to program notation or symbol from the given situation and interpret the given graph. A symbolic representation can be in the form of mathematical statements or notations, numerical symbols, or algebra [8]. Symbolic representation focuses on symbolic notation and includes the utilization of variables and formulas.

1. Discussion

Since there are various mathematical representations, students need to repeatedly learn how to solve problems with such mathematical representation types that aim to develop their ability to arrange problem-solving ideas. Students can use representation to find and create thought processes in communicating their mathematics ideas from an abstract concept into a concrete concept that enables them to understand the problem more easily [5].

It is better to give students the problem that its solution can use various types of representations in learning mathematics. Here are some examples of three mathematical representation types.

* 1. *Visual Representation*

A rectangle park has a size of 32 m in length and 24 m in width. Lamps will be installed around the park with a distance of 4 m between each lamp. The numbers of lamps needed are… (UN SMP 2015)

To solve such a problem, a visual representation can be used in the form of an illustration as follow.



**Figure 1.** Problem solving using

visual representation

From the problem, students will represent using an illustration of a rectangle. Then, because lamps are to be installed around the park, students will draw points to put each lamp. By ensuring the distance between each lamp, students can easily count the number of lamps surrounding the park.

* 1. *Verbal Representation*

If is a number between 6 and 9, then is between what two numbers? (TIMSS 2011)

To overcome the problem, a verbal representation can be used by explaining such sentences.

Because t is a number between 6 and 9, then the numbers are 7 and 8. The question is between what numbers is , and therefore 7 and 8 plus 5 becomes 12 and 13. After discovering the asked numbers, it is concluded that falls between 11 and 14.

* 1. *Symbolic Representation*

A study club has 86 members, and there are 14 more girls than boys. How many boys and how many girls are members of the club? (TIMSS 2011)

To overcome the problem, a symbolic representation can be used by supposing existing variables in the problem to create an equation.

From the problem, there are two variables, i.e., female members and male members. It can be symbolized or supposed for female members as a and male members as b. 86 is the total of members, hence, obtaining an equation of . Then, because female members are 14 more than male members, an equation of is obtained. From both equations, students can solve it using a two variables linear equation system as follow.

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In this symbolic representation, after students change the problem into mathematics form/symbols, students will determine the next step to be done. Therefore, students can find a suitable solution to the problem.

There are many ways to improve mathematical representation ability. Choices are to use the active learning method, the Realistic Mathematics Education (RME), and context-based learning or widely known as Contextual Teaching and Learning (CTL). Contextual learning gives students a chance to construct and represent mathematics ideas to a representation form so that they can develop mathematics understanding and problem-solving ability to solve mathematical problems [16].

There are also several things to be done that may support students’ representation process, that is:

1. Providing problems that can be solved using various types of representations.
2. Giving students a chance to solve the problem according to their representations.
3. Giving feedbacks towards students’ representation.
4. Encourage students to ask “What if?” and ask about other representations.
5. Conclusion

Mathematical representation ability is one of the essential abilities for students and one of the aims of mathematics learning in school. Representations that can be used to solve mathematical problems are verbal, visual, and symbolic representations. Representation is useful in helping students to solve problems and develop conceptual understanding. Representation is also useful as a means to communicate a student’s mathematics ideas to other students or teachers. In the in-class mathematics learning, it is better to provide students with a chance to train and develop their mathematical representation abilities. In mathematical representation development, it is crucial to consider students’ initial ability. Teachers’ role in explaining materials and providing problems is also vital to develop students’ mathematical representation ability.

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