How important is epistemology in learning mathematics?

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**Abstract**. Epistemology is one of the essential aspects of mathematics education. Personal views on epistemology affect how individual attitudes towards knowledge and achievement in learning mathematics. However, epistemology is only seen as a theory of truth of mathematical knowledge and proof of mathematics in its implementation in learning mathematics. Students do not understand that epistemology is 'knowing what to believe' rather than merely accepting mathematics as a 'finished product.' Therefore, a study of the importance of epistemology in mathematics learning needs to be done, to understand students about the epistemology of mathematics and its application in mathematics learning. With a review of the literature, this article attempts to explain students' beliefs in mathematical epistemology. The results of the study show that the students' perception of mathematics is mathematics as a result-oriented subject and is a static rule that cannot change.

1. Introduction

Epistemology is interesting educational research regarding the theory and concepts of knowledge, including mathematical knowledge. Epistemology is the "root" of philosophy that discusses the nature of knowledge and the justification of belief. The epistemological theory forms differences in personal views about knowledge and acquiring knowledge [1]. However, epistemology is often viewed as just a theory, discussing the history of scientists discovering knowledge so that it is rarely applied in mathematics learning. In fact, epistemology is necessary for learning mathematics, both for teachers and students who are directly involved.The first paragraph after a heading is not indented.

The use of the history of mathematics in mathematics learning is not something new. The history of mathematics is only used as humanizing mathematics in mathematics learning. History is regarded as a consistent whole, ready, stable, and limited knowledge ("finished product"). Furthermore, mathematics itself is also considered a "finished product." In studying the history of mathematics, teachers and students must be able to learn the interrelationship of forming new mathematical knowledge, the systems that are in it, and the things that support the formation of this knowledge. According to Gert [2], there is a strong consensus that the use of the history of mathematics should improve the quality of teaching mathematics. In other words, mathematics education has changed from a mathematics concept to mathematics learning. The NCTM standard states that the assessment of students 'beliefs about mathematics is an essential component of assessing students' overall mathematical knowledge. Exploring students' beliefs regarding the nature of knowledge and the learning process is vital in educational research. Furthermore, according to NCTM, belief is an essential factor in considering the impact of these beliefs on students' cognition and motivation [3].

Research on the importance of mathematical epistemology is undoubtedly not something new. This is shown by the large number of empirical research conducted to research or study the epistemological theory of mathematics and its implementation in mathematics education. However, there have not been many studies on the student's point of view and how students believe in the epistemology of mathematics.

1. Method

This article uses the literature review method, mathematical epistemology, mathematics proof, and epistemological beliefs of articles and journals. The study of the articles was obtained from the basic data of Google Scholar and ResearchGate. The keywords used are 'the definition of mathematical epistemology', 'the importance of epistemology in mathematics learning', 'mathematical proof', 'application of epistemology in mathematics learning', and 'epistemological beliefs.' The articles used in the literature review are following the research objectives, namely, to explain the need for a theoretical 'link' with its application in mathematics learning.

1. Discussion

## What is the definition of mathematical epistemology

According to Amer [3], epistemology is divided into three areas of study with three questions, namely: what are the limits of human knowledge? What is the source of human knowledge? Moreover, what is the nature of human knowledge? The question of what the limits of human knowledge are addresses whether there are questions that are impossible for humans to derive, gather evidence, or gather reasons so that they can be rationally justified. Furthermore, the question of what is the source of human knowledge addresses the sources of genuine knowledge, whether the source of knowledge comes from sensory experience or from pure reason. Finally, the question of the nature of human knowledge is related to conceptual analysis, namely, knowledge and truth, for example, what it means for someone to know something, concern with what one justifies in believing, and what one justifies. Anna & Stephen [4] explain in more detail that epistemology as a branch of philosophy that deals with scientific knowledge raises fundamental questions, namely: what is the origin of scientific knowledge? (empirical or rational); what are the criteria for the validity of scientific knowledge? (able to predict actual events, logical consistency); what are the characteristics of the scientific knowledge development process? (accumulation and continuity, scientific revolutions and discontinuities, or shifting and refinement in scientific programs). These questions can be used in general or specifically related to a particular domain of scientific knowledge such as mathematics. The epistemology of classical mathematics defines the position of epistemology as "rational reconstruction", that is, a description of the thought processes of mathematicians trying to communicate and justify their findings. The "context of justification" deals with scientific thought, while the "context of discovery" deals with the empirical domains of psychology, sociology and the history of knowledge.

## What is proof

All sciences confirm the results obtained and only a few sciences claim to prove these results, among them mathematics. The specialty of mathematics comes from its unique epistemology related to proving with a special technique called mathematical proof [5]. Evidence in mathematics is a communicative action that presents a solution to a problem (or the success of a procedure) created by the mathematician community [6]. As an action, evidence has a distinguished role based on the reader of the evidence, namely: (a) sceptical, having doubts about the theorem or conclusion of a proof; (b) have confidence in the conclusion of the evidence. In contrast to the previous opinion, Gallier [7] said there is no specific definition of evidence but it can be explained, evidence is a kind of deduction (derivation) that comes from a series of hypotheses (premises, axioms) to get a conclusion using some logical rules. Gallier [7] also explains that there are various levels of formality of evidence, namely: (1) evidence can be very informal: it uses a set of logical rules that are "loosely" defined, allowing for missing steps or premises; and (2) the evidence can be completely formal: it uses a very clear set of rules and premises, usually processed or created by programs called proof checkers and theorem provers. However, it is practically impossible to write formal evidence due to several factors including time consuming, difficult to read, difficult to write, and limited space to write it. So that we need "tools" in constructing and examining formal evidence, namely reasoning. The most important thing is to clearly understand the rules of reasoning used in making informal evidence. So that what is more emphasized in proof is formalizing the basic rules of reasoning used in natural deduction proof.

## Why is epistemology important in the learning and teaching of mathematics

After knowing what is the epistemology of mathematics and the relationship between epistemology and proof, the next question is why we should know epistemology? Will studying epistemology make it easier to understand mathematics? Does epistemology need to be taught in schools or just introduced? First, we will discuss mathematics first. Mathematics as a result of human intellectual activity, of course, goes through a process that is not short (has a long history) to produce a set of rules that are accepted today. Mathematical knowledge is not only explained by the state of mathematics that has become a deductively structured theory but also by the procedures that lead to the theory [8]. It is clear that learning mathematics is not just using or applying mathematical theories. Learning mathematics includes not only the "finished product" of mathematical activities but also understanding the implicit motivation, logical actions, and reflective processes of mathematicians who aim to construct meaning. In teaching mathematics, teachers must give students the opportunity to do mathematics (do mathematics). In other words, the "finished product" which is part of the communicated mathematical knowledge and the process of producing mathematical knowledge is equally important to be communicated in mathematics learning, especially from a didactic point of view. To view mathematics as a logically structured set of products and as a process of producing knowledge should be at the heart of mathematics teaching. The integration of history and epistemology in teaching and learning mathematics is a way that can lead to a better understanding of mathematics as a scientific discipline [8].

The proof is very important in learning and teaching mathematics. Reasoning and proving activities are at the heart of the process of forming mathematical sense and are important in student learning since elementary grades [9]. Accordingly, Gila & Hillsdale [10] stated that proof is an important characteristic of mathematics and is a "key" component in mathematics education. Balacheff [11] describes five epistemological positions entitled "Windows on epistemologies of mathematical proof", as follows: (1) mathematical proof as a type of universal proof and examples of good proof (mathematical proof as a universal and exemplary type of proof); (2) an approach that considers mathematical proof as a special nature (to approaches considering mathematical proof to be of an idiosyncratic nature); (3) mathematical proof is the essence of mathematics (considering mathematical proof at the core of mathematics); (4) tools needed by mathematics (applications) (as a tool needed by mathematics but that gets its meaning from applications); and (5) as autonomy in the realm of mathematics (being specific to mathematics as an autonomous field).

In detail, Balacheff [11] also describes each epistemological position. First, mathematical proof as a type of universal proof and good proof examples make it clear that the concept of proof focuses on increasing student understanding. The concept of proof does not only cover tasks related to mathematics but can be applied in all situations to reach conclusions and make decisions. Mathematics has made an extraordinary contribution to the development of the concept of proof. In addition, the evidence is the main argument, the result of human activity, so that students should be able to participate. The aim is to help students find their own concepts regarding mathematical justification. Second, an approach that considers mathematical proof to be idiosyncratic explains that a person's proof scheme consists of what is meant by ascertaining and persuading the person. Confirming and persuading is subjective and can vary from person to person, from civilization to civilization, and from generation to generation. Ultimately, a person's proof schemes are "idiosyncratic" and can vary from one field to another, or even within mathematics itself. Third, mathematical proof is the essence of mathematics, explaining that proof is the "heart" of mathematical thinking and deductive reasoning supports the process of proof, providing an example of the difference between mathematics and empirical science. At this point, Balacheff [11] presents the results of research carried out in London with the relatively low achievement of proof-constructing with more success in algebra than in geometry. Most students understand the general nature of valid evidence: students are better at recognizing valid arguments than constructing evidence, the concept of proof and their rules of evidence is crucial to student performance. Where student performance is independent of teacher characteristics but closely related to the number of hours spent learning mathematics, the explicit emphasis is placed on verification and student level (educational level or content familiarity). Teaching mathematical proof does not have to emphasize the form of proof but the meaning of proof in mathematical activities.

According to David Tall (Balacheff, [11] ), students' cognitive development needs to be considered so that evidence is presented in a meaningful form. Fourth, the tools needed by mathematics explain the potential and significant contribution through the communication of mathematical understanding. A mathematics curriculum that aims to demonstrate the real role of mathematical proof should present it as an indispensable tool rather than the core of the science. Fifth, as autonomy in the realm of mathematics discusses geometric facts, namely the theorem can only be accepted because it is systematized in a theory with complete autonomy from any verification or argumentation at the empirical level.

## How Do Students Believe in Mathematical Epistemology

Mathematical beliefs are a person's view of mathematics as required in a mathematical approach or task. Beliefs are important components that help create meaning and set relevant goals in mathematics learning. If belief influences the way students engage in mathematics learning and problem solving, then it is necessary to show what and how students believe in mathematics's epistemology. Muis [3] states that the belief in mathematics's epistemology includes views on the nature of mathematical knowledge, justification of mathematics knowledge, sources of mathematical knowledge, and ways of obtaining mathematical knowledge. Students' beliefs about mathematics as a scientific discipline are related to the perception of mathematics as a process-and-rule-oriented subject, a dynamic subject or a static subject, the perception of mathematicians and mathematics as a formal discipline.

Musi [3] presents several studies on students' beliefs about mathematical epistemology. The majority of research results indicate that students at all levels have beliefs that are not relevant. When asked questions about the certainty of mathematical knowledge, students believe that mathematical knowledge has not changed. Besides, students also believe that the goal of solving math problems is to find the right answer, even though they use and existence of mathematical evidence are used to support mathematical ideas. In other words, students must also understand the process and reasons for solving math problems. Students also believe that mathematical knowledge is passively transferred by mathematical figures, teachers, or textbook authors to be unable to learn mathematics through logic or reasoning. Furthermore, students believe that they are skilled in mathematics because of innate abilities (talents), the structure of mathematical knowledge is not related, and students do not believe that they can construct mathematical knowledge and solve problems independently.

## Implementation of epistemology in learning mathematics

The process of learning and teaching mathematics not only communicates the "finished product" of mathematical activities but also the process of producing knowledge. Students should be given the opportunity to understand implicit motivation, logical action (reasoning), and the reflective process in learning mathematical proof concepts. In addition, teaching mathematical evidence does not have to emphasize the form of evidence but the meaning of evidence in mathematical activities according to students' cognitive development. The epistemology of mathematics is important in learning and mathematics. Learning and building mathematics not only communicates the "finished product" of mathematical activities but also the process of producing knowledge. Students must be given the understanding to understand implicit motivation, logical action (reasoning), and the reflective process in learning mathematical proof concepts. Furthermore, mathematical evidence does not emphasize evidence but the meaning of evidence in mathematical activities according to students' cognitive development. The following are examples of implementing epistemology in mathematics learning.

1. Using math history

Epistemological theory claims that the development of mathematics is so basic that no conceptual change is understood. However, Teoplitz's theory provides guidance that the application of mathematics should allow teachers to establish their own meta-knowledge about mathematics [2]. This means that the teacher can determine for himself how mathematical knowledge will be known by students in learning mathematics. In addition, it is necessary to make a new understanding of the development of mathematics. The development of mathematics is not only an adaptation of concepts from general history (sociology, psychology, etc.) but also requires research on the learning process that will reduce the cultural element in the history of mathematics. This needs to be done in mathematics learning so that students have an understanding that mathematics from the process of finding knowledge is not a "finished product" from mathematicians and is static.

1. 'Seeing' student mistakes

Mathematics learning can be described as a process of "agreement" between teachers and students, where the teacher tries to apply certain procedures that may be dynamic [2]. Student errors are usually seen as a misconception, even though students "may" see other ideas in the material presented by the teacher than what the teacher has in mind. In this case, the teacher should pay more attention to the student's process of finding and understanding knowledge through informal means.

1. Communicating the activities to be carried out to determine the nature of knowledge, the use of learning contexts that utilize students' understanding (about mathematics) and push them to develop epistemological beliefs need to be carried out to facilitate the process of knowing of knowledge[12]
2. Increase epistemological knowledge in coaching learning so that the communication process is social and interactive. One important aspect of mathematical epistemology is the problem of how mathematical notations and symbols mean the interactive processes during learning and mathematics [13]. Students' mathematical knowledge is personal, context-bound, and is in the process of open development. Besides, students also have many and varied learning resources so that mathematics teachers must be aware of the specific epistemological status of students' mathematical knowledge. The teacher must be able to diagnose and analyze students' mathematical knowledge constructs and compare these constructs with what is available to study to vary the learning offerings accordingly.
3. Conclusions

Based on the description above, epistemology has an important role in improving the quality of learning mathematics. In short, epistemology is the process of knowing mathematical knowledge. In its implementation, epistemology theory requires a "connector" so that it can be applied in learning. As for how the example of the implementation of epistemology in mathematics learning can be done through: the use of the history of mathematics, compiling meta-knowledge, seeing the potential development of mathematics from student mistakes and creating nature-mathematics knowledge.

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