**The Development of High School Students’ Mathematics Justification Skill through *PMRI* Learning Activities**

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**Abstract**. *The main goal of this present study is a product, a sequence of learning activities which support the development of students’ mathematics justification skill. The product will be materialized by answering questions namely, (1) how does a form of learning activities based on PMRI could facilitate the development of students’ mathematics justification skill? And (2) how do designed learning activities based on PMRI could help the development of students’ mathematics justification skill? This present study used Research and Development type of study. The research design used here is ADDIE development model. This study conducted in three high schools in Sleman Region, DIY. Besides students, the mathematics teachers also involved. The results of this present study show that learning activities based on PMRI approach could facilitate the development of student’ mathematics justification skill. For every meeting, there was the number of students increased who participated in discussion, stated ideas and opinions, and convinced their finds what they did was the right thing. The learning activities based on PMRI could help the development of students’ mathematics justification skill by focusing on PMRI characteristics as one of approaches developed in the line of constructivism.*

# Research Background

Mathematics is often considered as a set of procedures, formulas which have to be remembered by heart so that one could solve mathematics problems well. Usually, students’ skill in solving mathematics problems is considered as a sign that he understands about the concept. However, this student in the future will experience difficulties to solve new problems and make connection to the previous concept he has been learnt, therefore, what is considered as a comprehension is only a part of memorization and results of drill given at school. Learning without understanding is not a new problem in education realm. Studies by [1] and [2] stated that learning without understanding has been a problem since 1930s and it has become a discussion subject and a research focus by psychologist and educators also stakeholders for years.

One way to enhance students’ mathematical understanding is by communication. Communication in mathematics can be in the form of oral, writing, formal, and informal. Through communicating, students reflect, clarify, and broaden their ideas and their understanding about relationship or relation and mathematical arguments [3]. Learn to argue about mathematical ideas is a fundamental subject in understanding mathematics and learning to think mathematically. NCTM also affirmed that communication, argumentation, justification skills are the basic things in learning mathematics in every level of education [4]. Students from any level of education should be able to communicate their

mathematical thinking, analyze others’ thinking, state appropriately mathematical language to present ideas, and develop also evaluate mathematical arguments and proofs.

Since mathematics is often stated in symbols, oral, and writing, communicating mathematical ideas is not always known as an important part in mathematics education [5]. In their research, [6] found that students are not required to speak about mathematics. Even though students’ works or solutions are mostly discussed together with their peers, it is only limited to strategies they used and steps they took in solving problems, if students are asked to represent and explain their answer, they reread what they have written. By explaining, we do not know why students used those strategies and how they were sure that those answers were right. In another words, this condition shut the possibility for teachers to know how students’ mathematical reasonings and how far students understand the subject also their misconception that probably happen in students’ mind. Students will only know procedures and ways utilized in solving problems and gaining results.

To create mathematics learning environment which can sharpen students’ reasoning and help teachers in monitoring students’ understanding, it needs a meaningful activity in mathematics discussion. This activity is known as justification. Justification, in practice provides bases, proofs, and reasons to convince others (or invite oneself) that a claim or a confirmation is true [7]. Justification is not only important for students, but also for teachers, where justification gives teachers to look at the development of mathematics understanding [8]. When students justify, they used their knowledge and reasonings to connect ideas or understand new things. This situation encourages teachers to understand students’ ideas and help them to improve or construct their knowledge by giving particular questions which bring them from only explaining problem solving strategies towards justifying, defending, and generalizing solutions strategies [9].

Studied about the importance of justification in teaching learning process in the classroom have been done by many researchers, for instance [10], [11],[12], and [13]. However, only few studies inserted the approach used in the learning process which supported students to do mathematics justification. Because of that, this study will take a *Pendidikan Matematika Realistik Indonesia (PMRI)* adapted from Realistic Mathematics Education (RME) from the Netherlands. Justification is considered fit to PMRI since justification is a part of interactivity one of PMRI principles [14].

As a part of interactivity, PMRI put communications as one of important parts in mathematics and mathematics education. Communication activity is then arranged in the form of a regulation or norm known as sociomathematical norms. Justification is one of communication forms surely in line with PMRI and could be realized by implementing sociomathematical norms. In this present study, sets of learning activities will be designed based on PMRI theory. By involving some high schools in Yogyakarta Special region, researchers will design sets of learning activities which facilitate the development of students’ mathematics justification skill by utilizing PMRI theory as a basis.

According to research background stated before, here are formulation of problems and research questions opposed, namely (1) how does the form of learning activities based on PMRI which could facilitate the development of students’ mathematics justification skill? and (2) how does the designed learning activities based on PMRI could help the development of students’ mathematics justification skill? As for the goal of this present study, generally, a product in the form of learning activities which supports the development of students’ mathematics justification skill. This product will answer research questions stated before.

# Research Method

The type of research conducted in this present study is Research and Development (R&D). R&D is a process or steps to develop a new product which have already been exist and can be accounted for. A product expected in this study are learning tools consisting of lesson plan and students’ worksheets. Based on PMRI which develop students’ justification skill. The design of this study is a developing model ADDIE. The ADDIE model is developed by Dick & Carry to design learning system. ADDIE consists of five phases, namely analysis, design, development, implementation, and evaluation [15].

The object of this study is learning tools, lesson plans and students’ worksheets based on PMRI and oriented to justification skill. The study will be conducted in three different high schools in Yogyakarta Special Region. Besides students, mathematics teachers who teach in the classroom will be involved. Collecting data technique used in this study is test and non-test. The test conducted aims at students’ mathematical justification skill. The results will be used to determine the effectiveness of lesson plan and students’ worksheet. The non-test instrument in this study consists of validating sheets, lesson tools, students’ response questionnaires, and observation sheets of learning compliance.

# Results and Discussions

Learning tools based on PMRI oriented to justification skill were developed using an ADDIE model. Generally, both teachers and students from the three-schools had similar characteristics which were elaborated as follow.

* 1. *Analyzing phase*
		1. *Competence analysis*

Mathematics learning competence used by these schools have referred to the 2013 curricula. This competency composes of main competence and basic competence which is then elaborated into indicators of competence achievements. In this study, there were different indicators for each school involved, namely perimeter and area of tetrahedron and triangle; surface area and volume of cube and cuboid; and surface area and volume for prism and pyramid.

* + 1. *Necessity analysis*

Based on interviews with mathematics teachers, they have already prepared mathematics lesson tools which facilitate students to construct their own knowledge. However, based on observations of learning in the classroom, students hardly got opportunities to find new concepts and construct their own understanding. Lesson plans only served as an administration formality of learning and have not been adjusted with students’ characteristics and necessities. The used of students’ worksheets in learning was also limited. Students’ worksheet which was usually used consisted of brief materials and sets of problems and have not provided activities engaging students to construct their own knowledge. During their lessons, students only received and listened materials from their teachers and barely showed enthusiasm since learning tended to be stiffed and boring.

Moreover, students were often noisy throughout learning and did not pay any attention towards materials from their teacher. When one of their classmates answering problems in front of the class, even came up with different strategies, students neither gave any responds nor present their own strategies. Students only focused on the final answer or numbers stated and did not notice the steps of procedures presented and also the reasons behind the answer.

* + 1. *Students’ characteristics analysis*

According to observation results during mathematics learning in the classroom, it was concluded that learning was still dominated by teachers’ activities therefore students tended only to listen the whole lessons. Students were passive during discussion sessions. When presenting groups’ works, students had difficulties explaining how they did the procedures altogether and convincing teachers and their classmates the reasons why their solutions was the correct one. Thus, students have not gained any opportunities to develop their justification skill.

Furthermore, based on Piaget theory, high school students are in the phase of operational formal which means that they have already been able to reason, think critically, think in higher level, use deductive hypothesis thinking, and arrange procedures systematically to solve problems. In another words, students have already been able to perform justification process towards mathematical problems.

* 1. *Designing phase*

Designing phase was divided into three parts, namely organizing lesson plans, students’ worksheet, and instruments for learning tools evaluation.

* + 1. *Organizing lesson plans*

Based on main competence, basic competence, and indicators, researchers designed lesson plans for there different schools and different materials.

**Table 1**. Learning activities design

|  |  |
| --- | --- |
| Phases | Activities |
| Introductions | Apperception, motivation, and learning goals |
| Using contexts | Students observe problems or real context given |
| Utilizing models | Students formulate models or charts or schemesas explorations results |
| Developing students’creativities | Students use their models to find solutions ofproblems |
| Supporting interactivities | Students are given opportunities to interact ingroups to solve problems |
| Making intertwinements | Students are able to intertwine concepts which are discussing and other relevant concepts whenneeded to solve problems |
| Closure | Reflections and evaluations |

Generally, all lesson plans followed Table 1 as a design of learning activities. Each lesson had its design and usually conducted in 2 x 40 minutes.

* + 1. *Organizing students’ worksheets*

Students’ worksheets frameworks consisted of three parts, namely first part, main part, and last part. The first part composed of cover, identity page, foreword, table of contents, manual instructions, concept maps, and competences pages. The main part consisted of four material for each which was separated into basic competence, indicators, learning goals, manual instructions, and activities based on PMRI theory. The last part was for references and brief profile of researchers.

* + 1. *Organizing instruments for learning tools evaluation*

From this process was yielded evaluation sheets for lesson plans, students’ worksheets, students’ responses questionnaire, and mathematics justification skill test instruments.

* 1. *Developing phase*

In this phase the frameworks of products were materialized into ready made products. The products then were validated by two validators and revised based on suggestions and recommendations given.

* 1. *Implementing phase*

Lesson tools which were already revised and stated to be decent then tested at schools. In the first meeting, it was found that students faced difficulties adapting with new learning environment created by researchers and teachers. For example, when they were asked to work in groups, students barely

conducted discussion with each other, they tended to divide the work so each member of groups solve their own parts and, in the end, they gather all results without having need to discuss the answers. It also happened when teachers asked one of the groups to present their solution in front of the class, the one who presented the answer was the only one who seemed to understand the problem well while the other members of group kept silent. The way they presented the answer was only by reread what they wrote in their worksheets. Class discussion did not run smoothly according to the plan since students did not any questions related to the solution or procedures because they had the same number as a result. Teachers tried to support the development of students’ justification skill by asking some questions and motivation to broader their curiosity.

On the second meeting, students started to be braver than before. They started to explain how they came with their answers. Teachers responded their answers and gave appreciation. Group discussion in the second meeting was more active than the first one. Teachers monitored the discussion and guided students to construct their own knowledge by productively asking questions. Generally, this set of learning activities developed better and better in which students were comfortable in communicating their ideas or arguments to others. They also started to question other group solutions including procedures or steps and shared their own solution or thinking even though both came up with the same results. In the end of materials, teachers conducted a test to evaluate students’ mathematics justification skill.

* 1. *Evaluating phase*

Evaluation phases was the last phase after implementing conducted or testing developed lesson tools. Evaluation phase was done to repair lesson tools based on suggestions and recommendations or findings during implementation phase. In this phase, all lesson plans and students’ worksheets were validated so that they were valid and decent to utilize in the learning. As a result, in this study, mathematics justification instrument, student’s response questionnaire, learning implementation observation sheets were categorized valid and suitable to be tested.

The used of lesson plans, students’ worksheets, and justification evaluation instrument was solely aimed at developing students’ mathematics learning becomes better. Lesson plans, students’ worksheets, and justification evaluation instrument have already been through validation process and revisions. Validations involved two validators, an expert and a mathematics teacher. All lesson plans in this study were categorized valid and proper to be used in the classroom. So that the students’ worksheets, have been validated and got the average score categorized valid and proper to be used.

Students’ justification skill was assessed with five aspects from assessing rubric develop by Cioe et al (2015), namely calculation, labels, evidence, answer the questions, and reason why. Based on the result of students’ justification skill, generally, almost students have been able to develop their own justification skill. Even though they still needed helps and encouragements from teachers, students have strived to develop their justification skill potency. The final test which was distributed among students also promoted their justification skill in learning

Through the set of learning activities, it could be seen that PMRI theory helped facilitating the development of students’ mathematics justification skill. In every meeting we could see the increased number of students who participated in discussion, stated ideas and opinions, asked questions, convinced their peers that what they did was correct. Techers’ supports and helps in the form of questions gave significant influence in developing students’ justification skill. Students became more confident by their answers without a need to confirm them first to the teachers. Students also became more open to the critics and ideas from their classmates. Teachers was no longer solely knowledge sources for students and a place to answer all questions since students were given opportunities to find information and process the information to solve problems.

The designed learning activities based on PMRI theory could help develop students’ mathematical justification skill using characteristics of PMRI as one of learning approaches which develops in line with constructivism realm. It is not a new thing for teachers about learning mathematics using PMRI

theory. However, practices on the fields showed that not all PMRI characteristics were develop or implemented in each meeting. In designing lesson plans and students’ worksheet based on early observations and analysis conducted in three different high school in Sleman, Yogyakarta, researchers mainly focused on all elements of PMRI theory and how it should be implemented in each meeting. It was confirmed that PMRI theory gave very good impact in teaching learning mathematics generally and developing students’ justification skill particularly. In this study, teachers’ role also played as a significant part in developing students’ skill. They served as facilitators in the classroom and encouraged construction of discussion activities among students which in fact had a quite big influence. In line with the implementation of PMRI characteristics in learning, students’ justification skill developed well. This is evidenced by the result of students’ justification skill test which showed that most students have done justification in solving problems.

# Conclusions and Recommendations

* 1. *Conclusions*

From this present study, it can be concluded that

* Learning activities based on PMRI theory could facilitate the development of students’ mathematics justification skill. In every meeting was found the increased number of students participating in discussions, stating ideas and opinions, asking questions, and convincing their classmates that their solutions were correct.
* Learning activities based on PMRI theory could help the development of students’ mathematics justification skill by prioritizing PMRI characteristics as one of learning approaches which has developed in line with constructivism realm.
	1. *Recommendations*

This present study can be continued in longer time and in different level of education, for example in senior high school and college.

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