Application of model methods to improve students representation in solving mathematical word problems

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**Abstract**. Learning should help students to master problem-solving skills. Mathematical problem-solving skills can be improved using word problems. To be able to master problem-solving skills, students need to master mathematical representation skills. Mathematical representation skills are used by students to express ideas and their relationship from the word problems given. Also, mathematical representation skills will help students know the relationship between mathematical objects in the word problems. To improve mathematical representation skills, teachers need to design lessons that guide students in modelling mathematical word problems into concrete objects and their relationships. A model method is an approach that helps students to model mathematical word problems into real objects and their relationships. The model method can improve student's mathematical representation skills so that students can master mathematical problem-solving skills because students are accustomed to translating word problems into mathematical symbols and relations.

1. Introduction

Mathematical problem-solving skills are essential as a goal of learning mathematics [1]. In Indonesia especially mentioned that students must have the ability to solve problems in daily life and use the right strategies to solve real problems [2]. Mathematical representation skill needs to developed so that students can solve word problems [3]. According to Wilkie and Ayalon mathematical representation skills as the skills to model information in one context, such as real-life problems represented in value tables, and related to students' ability to use it [4]. They also define that mathematical representation skills are defined as generalizing patterns and making algebraic rules [4]. Furthermore, mathematical representation skills are defined as key in learning mathematics [5].

Mathematical representation is a skill that students use to master mathematics that includes solving problems, making connections, understanding various types of mathematical ideas, communicating ideas, and explaining these ideas [6]. Students can find and determine ways of thinking in communicate mathematical ideas from abstract objects to concrete objects so that they can understand mathematical problems easier with mathematical representation skills [7]. Saifiyah and Retnawati offer some aspects of mathematical representation skills include pictures, graphs, tables, equations, mathematical symbols, words, and writing [6]. In their research, they describe some difficulties that students faced in developing mathematical representation skills, including the difficulties of representing mathematical symbols into images, representing symbols into symbols in other forms, representing text into symbol form, representing text into images, and representing questions as symbols or pictures. In their research, they describe the importance of using symbols and pictures in the learning process that can encourage students to master mathematical representation skills [6].

Kaur proposed model methods as an approach that could guide students to describe mathematical objects concretely [8]. The model method is an approach developed by the Singapore ministry of education in 1980 and has become more widely known because the 2003 Trends in International Mathematics and Science Study (TIMSS) Singapore assessment showed very good results [9]. Bautista also adds that the use of the model method has also shown good results in the Program for International Student Assessment (PISA) assessment [9]. In a book entitled The Singapore model method for the learning of mathematics, it explains the use of the method model in four types of learning, specifically mathematical operations, fractions, ratios, and percentages [10]. With learning outcomes that are considered successful, many countries have begun to apply the model method approach (Bautista, 2015), so that there are many studies on the use of model methods adapted from Singapore.

The effectiveness of the model method needs to be studied to understand in-depth how it relates to mathematical representation skills. This article is aimed to describe the effectiveness of using the model method to improve student's mathematical representation skills in solving mathematical word problems. Using the literature review method, researchers refer to related international journals to study research results related to model methods, student's representation skills, and word problems.

1. Results and discussion

Mathematical problem-solving skills are essential as a goal of learning mathematics [1]. Problem-solving skill is an ability to understand and solve problems for which the solution cannot be found directly [11]. The problem-solving process includes exploring and understanding problems, representing and formulating problems, planning and executing problems, and monitoring and reflecting on the results obtained [11].

In PISA, problem-solving skills are measured through the ability to solve word problems [11]. Word problems are problems that have the context of daily life [12]. In Indonesia, students must have the ability to solve problems in daily life and use the right strategies to solve real problems [2]. Some types of student difficulties in solving word problems are the inability to read and understand the situations contained in the problems, such as comparing, changing, combining addition and subtraction problems, translating problems into mathematical equations, and doing calculations [13]. A study conducted by Newton shows that students have difficulty solving fractions, conceptual difficulties, and problem-solving procedures [14]. So that for students who have difficulty solving problems of fraction material story problems, intensive learning support is needed [15].

Word problems cannot be solved with just conceptual knowledge [16]. Students have to read the whole problem to identify missing information, make plans to find missing information, and perform calculations to find missing information [17]. Word problems must conclude the situation that relates to students' life so that students can identify the problems according to their understanding, find solutions, and convey these solutions both verbally and orally [18]. Word problems can be solved using a structured process in which students are taught to describe mathematical objects and various problem structures [19].

Several studies have stated that student's ability to represent problems in mathematical form and identify appropriate calculations determines student success in solving problems rather than numeracy [20]. Word problems do not give students a mathematical equation so that students need a complicated process to solve word problems, including reading questions, understanding questions, converting information in questions into mathematical equations, doing calculations, changing the calculation results back into the context of the problem given and doing an evaluation of the processes that have been carried out [21]. Student's ability to solve word problems does not only depend on the results of the correct answers but includes the ability to understand questions and master complicated problems, such as planning, conducting, and evaluating [22]. Besides, Abdullah, et.al added that students need to master mathematical representation skills [22]. Students must understand the symbols used, the language, and the visualization of the objects to solve mathematical word problems. Jitendra, et.al added that students will become experts in problem-solving if they can reproduce the structure of the problem into organized diagrams or graphs [23].

Mathematical representation skills are necessary to teach and learn mathematics [5]. Representation is the ability to describe phenomena and processes that are immediately visible or invisible, as well as their relationships [24]. Umbara, et.al states that mathematical representation is changing one form of an object into another form in mathematics [25]. Mathematical representations can be influenced by the way students perceive teachers, peers, and learning content such as textbooks [26]. Mathematical representations are used by students to come up with ideas and a deep understanding of the problems [27]. Lesh, Post & Behr divided mathematical representation skills into five types, namely real-world representation, concrete representation, arithmetic symbol representation, language representation, and image, or graphic representation [28]. Furthermore, Milrad states that the three types of representation are types of representation that have a higher level than the other two components, specifically [29]:

1. Language representation skills. Language representation skills are used to observe a mathematical object and their relationship into a verbal form or oral form.
2. Image or graphic representation skills. Image or graphic representation skills are used to translate mathematical problems into an image or graphic form.
3. Arithmetic symbol representation skills. Arithmetic symbol representation skills are used to translate mathematical problems into an arithmetic symbol.

Every student has a different learning model [30]. Therefore, teachers should use an appropriate approach to improve mathematical representation skills [31]. NCTM sets a standard of representation that is expected to be mastered by students during learning at school, specifically [32]:

1. Creating and using representations to recognize, take a note or record, and communicate mathematical ideas.
2. Select, apply, and translate between mathematical representations to solve problems.
3. Using representations to model and interpret physical, social, and mathematical phenomena.

Mathematical representation skills can be enhanced using the model of problems that gave to students and relate it to student’s conceptual knowledge [33]. Here an example of word problems that can be solved effectively when students using mathematical representation.

”*What will happen to the area of rectangular if the length and the wide is doubled*?”

Students can determine the area of the rectangular after the length and the wide is doubled using substitution methods like shown in table 1.

Table 1. Substitution method to solve the problem

|  |
| --- |
| The area of rectangular (A)  If the length and the wide is doubled then the area of the new rectangular must be as shown as bellow  So the area of the new rectangular is four-time the old one. |

Table 1 shows the problem solving with the substitution method, in table 2 below shows the problem solving with the model method recommended by NCTM [32]

Table 2. Using model method to solve the problem

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| |  | | --- | |  |   Area of rectangle before the wide and length is doubled   |  |  | | --- | --- | |  |  | |  |  |   At the picture above we can see that the area of new rectangular is four times. |

Mathematical representation skills can be developed by providing opportunities for students to use concrete object representations in mathematics learning [34]. Mathematical representation abilities can be developed by providing problem models to students in the form of symbols, pictures, diagrams, schemes, and paradigmatic solutions [35]. Marwan and Duksri add that students who have low representation abilities are caused because students are not used to using representation as a step in problem-solving [36]. Thus, to develop mathematical representation skills, the teacher needs to design an approach that guides students to represent mathematical problems into a model of images, graphs, symbols, diagrams, or schemes. One approach that can be used is the model method. The model method will help students to visualize mathematical objects and the relationships between objects and various problem structures [37].

The model method is one of the approaches used in the Singapore curriculum to provide mathematics learning in elementary schools [38]. A model method is an approach that has similarities to schema-based instruction (SBI) [8]. Students can describe some or all of the objects in the problem concretely easier when the teacher uses the model method approach [8]. Morin et.al stated that the use of the model method is effective to assist elementary school students in learning [39]. The images used to visualize are not completely the same as the objects in the story but describe the quantity and relations between objects in the problem [20]. The method model does not have a theoretical guide regarding the learning sequence [8]. This means that the model method only uses three elements, namely the concrete-image-abstract approach which includes object design and the main teaching pedagogy of mathematics [8]. However, in the learning process using the model method there is a process of understanding problems, selecting variables, forming mathematical models, solving problems, and interpreting the results of problem-solving into concrete problems [40]. When students use the model method, the addition and subtraction operations are no longer computational operations but become relations between concrete objects that have been visualized [41]. The use of pictures and real examples can make it easier for students to solve story problems [20].

Students who solve problems using the model method can solve problems systematically [20]. Also, the use of the model method can improve student's problem-solving skills because with the model method learning, students will understand the concept of mathematics as a relationship between related objects and not just computation [42]. Research conducted by Sagirli also shows the same thing [43]. In his research, Sagirli suggests that using the model method can improve student's understanding and problem-solving skills [43]. This is because by using the model method, students can see mathematical problems to be more concrete so that it is easier to develop a thinking process to find solutions and help students to interpret them more easily [43]. Table 3 below shows an example of applying the model method described by Kaur [8].

Table 3. Solving the problem using model method

|  |
| --- |
| Based on the example below  David has four toy cars and Ali has six toy cars. What is the amount of their toy cars together?  In grade one, students use concrete objects (or cut-and-stick pictures) to form the two groups of toy cars that each child has.    Then the students add 4 to 6 to the total of the toys.  Students will write the arithmetic form 4 + 6 = 10 as problem-solving process.  In second grade, students draw pictures to represent the object or the toy cars of both children has.    The problem's model can be described as a whole of two parts. Students will add all sections to find the total number.    At the end of the problem solving process, student will conclude that David and Ali have 10 toy cars. |

According to Ng and Lee by using the model method, students are able to construct mathematical objects and their relationships well so that students are able to make appropriate arithmetic plans and calculations to solve problems [20].

From the explanation above, it can be concluded that the model method can help students to improve their mathematical representation skills. The model method approach will guide students to represent the objects in the story problem into concrete objects and to know the relations between the objects to then determine the problem-solving procedure.

1. Conclusion

Mathematical representation skills are an important skill in mathematical problem-solving. Mathematical representation skills can help students to understand problems and determine relationships between objects. Besides, mathematical representations skills help students to communicate ideas of the problem and the ideas of the problem-solving procedure. One approach that can improve a student's mathematical representation skills is the model method. By using the model method approach, students will be trained to represent problems in the form of pictures, graphs, or schemes. Therefore students will be able to master mathematical representation skills through mathematics learning.

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