Introducing an arithmetic sequence using realistic mathematics education principles

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**Abstract.** An arithmetic sequence is one of algebra topics introduced for grade VIII students in Indonesian mathematics curriculum. The question is how do we design a learning material on this topic so that students understand it meaningfully? This study therefore aims to design a learning material for introducing the arithmetic sequence. To do this, we used a design research method, particularly the phase of a preliminary design with the following three steps. First, we carried out a literature study on the theory of Realistic Mathematics Education (RME) and a limited observation on Indonesian mathematics textbooks on the topic of arithmetic sequence. Second, we designed a learning material for the topic of the arithmetic sequence according to the RME theory and the Indonesian mathematics textbooks. The RME theory was used because it provides guidance for designing meaningful learning activities. Finally, we made predictions of students’ thinking process to produce a ready-use learning material for the teaching experiment. The learning material, as the result of this study, included a learning sequence and corresponding predictions of students’ thinking process for the topic of arithmetic sequence. We consider that the learning material is more meaningful to be implemented in the learning and teaching process.

1.Introduction

Algebra as one of branches of mathematics is considered to be a difficult domain for most of junior high school students not only in Indonesia, but also in other countries [1–5]. Difficult algebra topics for most of the students include linear equations and inequality in one variable, arithmetic operations on algebraic expressions, number patterns, and number sequences—including arithmetic sequence [5–6]. This difficulty should be overcome, for instance, by designing a meaningful learning sequence for students through a cyclic design and testing process.

Previous studies on the design of learning materials on the topic of arithmetic sequence for junior high school students in Indonesian contexts mainly focused on the design of learning materials according to general learning models, such as, according to discovery learning [7], problem based learning [8], and scientific learning [9]. In these studies, however, the design of learning materials is not conducted according to a domain specific theory of didactic of mathematics. As a result, the learning designs often do not consider the relationship between the development of student mathematical thinking and the didactic of mathematics. The question is how a learning material for the topic of arithmetic sequence is designed so as to develop student mathematical thinking in a meaningful manner.

To address this question, we carried out the present study which aims to design an algebra learning material on the topic of arithmetic sequence according to the theory of realistic mathematics education (RME). This domain specific theory in mathematics education is used because it has been successful in the Netherlands and other countries for teaching mathematics in a meaningful manner [10]. For the design process, we propose to use three didactical principles of the RME theory, including the reality, the level, and the intertwinement principles [11]. The reality principle suggests that mathematics learning process should start from meaningful and rich contextual problems that need to be mathematized [11-12]. Mathematization concerns an activity of transforming contextual problems into symbolic problems, of reorganizing and of reconstructing the symbolic problems within the world of mathematics [11-13]. The level principle suggests that in the learning mathematics process students pass different levels of understanding: from context-related situations to a more abstract level of relationship between concepts and strategies [11]. The intertwinement principle suggests that mathematical content domains such as number, algebra and geometry should be integrated rather than as isolated mathematical domains in the learning and teaching process [11-12].

2. Methods

To design a learning material for the topic of arithmetic sequence for grade VIII students, we used a design research method particularly the preliminary design phase with the following three steps [14-18]. First, we conducted a literature study on the theory of Realistic Mathematics Education (RME), relevant research results, and a limited observation on Indonesian mathematics textbooks on the topic of arithmetic sequence [19-20]. Second, we designed a learning material for the topic of the arithmetic sequence according to the RME theory and the Indonesian mathematics textbooks. The RME theory was used because it provides guidance for designing meaningful learning materials. Third, we made predictions of students’ thinking process for the designed learning material. Thus, the results of these three steps include the sequence of learning and its corresponding predictions of the student thinking process.

3.Results and Discussions

This section presents the results of the design of the learning material for introducing the topic of arithmetic sequence. The results include a learning sequence and its corresponding predictions of student thinking. The learning sequence consists of three activities: Finding relations between a pattern and a sequence of numbers; finding a formula for an arithmetic sequence; and problem solving on problems related to arithmetic sequence.

3.1 Finding relations between a pattern and a sequence of numbers

The first learning activity concerns finding relations between a pattern and a sequence of numbers. The learning starts from an activity of finding the number of dots according to each figural pattern, and of predicting the number of dots for a next pattern. The numbers of dots for the given patterns have a specific property, namely the difference between two consecutive numbers is constant, which is deliberately used for introducing the arithmetic sequence. Next, after students get used to see the relations between patterns and numbers, students are expected to be able to predict a next number if they are given a sequence of numbers. Table 1 presents typical tasks for the first learning activity.

For the Task 1, we predict that students are able to determine a next dot pattern and the corresponding number of dots. So, the pattern of the corresponding number of dots is the following:

1 = 1

3 = **1** + 2

5 = **3** + 2

7 = **5** + 2

By obseving this pattern, students are expected to see that the number of dots for a next term is equal to number of dots of a previous term plus two. Therefore, the next term must be 7 + 2 = 9.

Table 1. Typical tasks for the first learning activity

|  |
| --- |
| Task 1. Contnue the following dot pattern and find number of dots for a next pattern. |
| Task 2. Continue the following number patterns. Find next three numbers for each sequence of numbers.1. 1, 4, 7, 10, 13, …, …, …
2. 27, 23, 19, 15, …, …, …
 |

For the Task 2a, we predict that students are able to determine next three numbers. If students encounter difficulties seeing the pattern, students are expected to use a dot pattern to draw each given number. So, similar to the case of Task 1, they are expected to observe the following pattern:

1 = 1

4 = 1 + 3

7 = 4 + 3

10 = 7 + 3

13 = 10 + 3

The pattern is that a next number is equal to the previous number plus three. In this way, the next three numbers must be: 16, 19, and 22. For the case of Task 2b, students are also expected to be able to find next three numbers similar to the case of Task 2a. From the first learning activity, with a teacher guidance, students are expected to recognize this specific sequence of numbers, having a constant term between two consecutive terms, which is so-called an arithmetic sequence.

From the perspective of the RME theory, the first learning activity applies the reality principle, that is, using the context of dot patterns. In our view, the dot pattern is realistic for grade VIII students because it is meaningful in their mind [11-12; 17-18].

3.2 Finding a formula for an arithmetic sequence

The second learning activity uses the experience of the first activity to find a formula for determining a term of the given sequence. Table 2 presents typical tasks given in the second learning activity. For the Task 3, we expect students to find a formula for the *n*th term by observing the pattern of the sequence of numbers as the following:

2 = 2

5 = 2 + 3

8 = 5 + 3 = 2 + 2 x 3

11 = 8 + 3 = 2 + 3 x 3

14 = 11 + 3 = 2 + 4 x 3.

Based on the above pattern, students are expected to find that the *n*th term of the sequence can be written as $T\_{n}=2+\left(n-1\right)×3=3n-1$. Therefore, the 100th term is $T\_{100}=3.100-1=299$. In a similar manner, students are expected to find a formula for the *n*th term of the arithmetic sequence in the Task 4.

Table 2. Typical tasks for the second learning activity

|  |
| --- |
| Task 3. Find the 100th term for the following sequence (Hint: Find the formula for the *n*th term)2, 5, 8, 11, 14, … |
| Task 4. Find the formula for *n*th term for the following sequence5, 9, 13, 17, 21, … |

From the second learning activity, with a teacher guidance, we also expect students to do a generalization when finding a formula for the *n*th term of an arithmetic sequence. By observing examples in the previous activities, an arithmetic sequence can be written as follows:

*a*, *a + b, a +*2*b, a +* 3*b, a +* 4*b, …, a +* (*n*-1)*b.*

where the first term $a$ and the constant difference between two terms is $b$. Therefore, the formula for the *n*th term of an arithmetic sequence can be written as $T\_{n}=a+\left(n-1\right)b$.

 In the light of the RME theory, the second learning activity has applied the level principle, in which students used a previous learning activity experience for finding a pattern and formula in the second learning activity [11-12; 17-18]. The intertwinement principle is also used because the process of finding and using algebraic formulas is an example of the relation between arithmetic and algebra [2-3, 5, 17-18].

3.3 Problem solving on arithmetic sequence

In the third learning activity students are expected to be able to use the prevous learning experiences and knowledge in the problem solving process. Table 3 presents typical tasks for the third learning activity.

Table 3. Typical tasks for the third learning activity

|  |
| --- |
| Task 5. Given the following arithmetic sequence:4, 11, 18, 25, 32, 39, …Investigate whether 2020 is included in the sequence or not. Explain your answer! |
| Task 6. Three consecutive numbers form an arithmetic sequence. The sum of these numbers are 21, and the multiplication of the numbers are 315. Find the three numbers. Explain your answer! |

For the Task 5, by using previous learning activities, students are predicted to be able to find the formula for the *n*th term of the sequence, namely $T\_{n}=7n-3$. Next, students are expected to be able to solve the linear equation in one variable $7n-3=2020$. By solving this equation, students can conclude that 2020 is within the sequence.

For the Task 6, students probably will at first find it difficult to solve this problem. This dificulty may arise, for instance, because students translate the task into a system of equations $a+\left(a+b\right)+\left(a+2b\right)=21$ and $a\left(a+b\right)\left(a+2b\right)=315$. As the system involves a polynomial equation, it will be difficult for junior high school students as they do not know yet how to solve such equation. By a teacher guidance, this task is expected to be solved by the students. For instance, by translating the task into an easier system of equations, namely $\left(a-b\right)+a+\left(a+b\right)=21⇔a=7$ and $\left(7-b\right)\left(7+b\right)=45$.

From the perspective of the RME theory, the third learning activity uses the intertwinement principle, in which students should use their repertoire of mathematics knowledge: not only on numbers, but also on algebra [12]. The ability to translate a contextual problem into a symbolic mathematics problem concerns horizontal mathematization; and the ability to solve the symbolic problem concerns vertical mathematization [3, 11-13].

4.Conclusion

Based on the results and discussion, we draw the following conclusions. The designed learning material for introducing arithmetic sequence includes a learning sequence and its corresponding predictions of student thinking process. The learning sequence consists of three activities: Finding relations between a pattern and a sequence of numbers; finding a formula for an arithmetic sequence; and problem solving on arithmetic sequences. The context used in the learning sequence is mainly numbers, which is considered to be meaningful for junior high school students. Three principles of the RME theory, i.e., the reality principle, the level principle, and the intertwinement principle, play an important role in the design process and in predicting student thinking process. Considering the promising results of the design, for a future research, we do wonder if the learning sequence for introducing an arithmetic sequence can be implemented in the learning and teaching process of mathematics.

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