Measure of Central Tendency

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**Abstract**. *The measure of central tendency is defined as a uniqe value that can be considered at the most representative from collected data and it can be determined through the mean, midrange, median dan mode. The properties of each measure of central tendency in the metric space will be discussed. Then, it will be shown that the mean minimizes the total error square in*  *space. Midrange minimizes maximum deviation in space. The median minimizes the sum of absolute deviation in spaces. The mode minimizes in the discrete metric. This study is a new discovery that shows that there relationship between statistics and functional analysis.*

1. Introduction

The measure of central tendency is defined as statistics that identify a single value as representatives of all data or in other words The measure of central tendency is the single most distinctive or representative value of a data. It is a "number" used to describe the description aspects of a data. In addition to being used to describe the description aspects of a data, The measure of central tendency is also useful as a estimator.

This Paper will analyze the properties or characteristics of the average value (mean), midrange, median and mode as one measure of the centrability of data from the glasses of functional analysis especially on the metric space. In addition to see if there is a relationship between statistics and functional analysis.

1. Main Results
   * 1. Mean

An arithmetic mean or an average value is the sum of all observation values divided by the number of observations. Denoted by x ̅ (pronounced x bar) and defined by:

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* + 1. A properties of mean. The arithmetic mean has two special properties, namely as an unbiased estimator and is a solution that minimizes the total error square. The arithmetic mean would be that makes minimizes. With S = {s, s,..., s} is a constant vector that is in the space. When in space and having vector x, we will look for something that causes the total error square or deviation of the minimum quadrat, then vector S the number.
  1. *Midrange*

According to Mario Triola [10], the midrange is the middle value between the maximum value and the minimum value of a data. Midrange is defined as follows:

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With is the smallest or minimum value of a data and is the largest or maximum value of a data.

* + 1. A properties of midrange. Review the function of maximum deviation In space . Then f (s) function will reach the minimum value when s value equals , with . Number which causes the function will reach this minimum value as a midrange. In other words, when then is the smallest possible value among the max value with s ∈ R .

*Proof*. When and suppose .Then,

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As a result, when then is the minimum value of the maximum value. We can see the illustration in Figure 1 below*.*

Picture 1: Ilustration of function when .

e.g :

*M=*

*m=*

But is it true that is the minimum value of the maximum value? To answer this question, will be proven if , then i.e. .

*Proof*. Let us first consider the case, if , then

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So when , it is proven .

We can see illustration of Case 1 in Figure 2 below,

Picture 2 : Ilustration function when .

s

Similary if then .

This proof at the same time asserted that is a midrange number minimizing the function .

* 1. *Median*

Samples are arranged in order of smallest data up to the largest data or vice versa, hence the median is defined as follows:

* + 1. A properties of median.. Review which is the distance between x and s. We will determine the s point so that the total distance between x and s is the minimum and it is suspected that s value is median. It would be proved that the median is a solution that minimizes the sum of absolute deviation, this proof will be different when n odd and when n is even.

*Proof*. Let us first consider the case where n is odd. Without loss of generality, we may assume that x is increasing. Our task is to show that is minimized at , that is,

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for every s ∈ . To do so, observe that if dengan for some δ > 0, then

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unknown , as a result

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Similarly, if with for some δ > 0, then

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This proves the case where n is odd.

The proof of the case where n is even is similar, we will examine two cases, i.e. the first case, the number *s is at intervals* or and the second case, number is located outside the interval or .

*Case 1*, e.g .

To prove that all numbers that are in interval will minimize we will examine any number that is at intervals . Suppose the number which is at intervals , but that number is not the median of an even data..

So that the absolute deviation is

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For the number which is greater than the median , it will be obtained the same absolute deviation of the same as the number which is smaller than the median .

For equals to the median of an even data or , the absolute deviation, we have

So

That is, all numbers located at intervals , the sum of absolute deviation will be the same as the sum of the median absolute deviation.

Case 2, e.g .

In case 2 there are two possibilities, such as: if with and Then,

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The first possibility of this indicates that if , then

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Similary if with dan Then,

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Based on the above evidence for n odd and for n even, it can be concluded that the median correctly minimizes the sum of absolute deviation in the space, in other words Reach the minimum when the S value equals the median value.

* 1. *Mode*

Mode is the data that appears most frequently or data with the greatest frequency. We can find three possibilities of the mode of the data set: If there are two data that appear most frequently or have the same largest frequency, then it is said that the data has two mode or called Bimode. If there are more than two data that appear most frequently or have the same largest frequency, it is referred to as multimode. If none of the data is repeated, it is said that the data has no mode.

* + 1. A properties of mode. Mode minimize on discrete metrics. Review the function . Then the function will reach the minimum value when value equals , where is the mode. This proof will be divided into three cases, namely if there is one mode, two modes and no mode in the data set..

*case 1*, there is one mode.

*Proof*. Suppose is the mode that appears as much as . Will be proven reaches a minimizes when On discrete metric, if then and if then . Unknown appears as much as , so

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So when then .

Suppose , apperas as much as . Meanwhile, the mode is the data that appears most often, so that surely the other data has a frequency smaller than frequency mode. Then

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Unknown , consequently

So when , then

It is evident that the mode minimizes on discrete metrics, since any value must have a smaller frequency than the mode frequency.

*Case 2*, there is more than one mode.

*Proof*. Suppose there are two modes, with the same frequency that is . Will be proven reaches the minimizes when and

for

With mode, then

Similary for **.**

Thus we can conclude that meaning true if or minimizes on discrete metric, because any value must have a frequency smaller than the frequency of the mode.

*Case 3*, no mode.

In this case, all numbers minimize . But since all numbers are not special, we can say if there is no mode in the single data that none of them is repeated or all numbers are mode.

From glasses to functional analysis, it turns out that the mode has properties minimizing on discrete metrics.

1. Conclusion

The measure of central tendency is defined as a statistical measure that identifies a single value as a representative of all data. This measure aims to provide accurate descriptions of all data. In other words, the size of the data concentration is the most distinctive or representative single value of the data. It is a "number" used to describe the description aspects of a data.

To be concluded that:

1. The mean minimizes the total error square in space.
2. Midrange minimizes maximum deviation in space.
3. The median minimizes the sum of absolute deviation in spaces.
4. The mode minimizes in the discrete metric.

This study is a new discovery that shows that there relationship between statistics and functional analysis.

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